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Advanced Computational Methods for Optimization of Non-Periodic Inspection Intervals for Aging Infrastructure

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This report for the project titled 'Advanced Computational Methods for Optimization of Non-Periodic Inspection Intervals for Aging Infrastructure' proposes an approach for optimization of non-periodic inspection scheme on a finite time horizon for a multi-component safety-critical system. The system consists of several components, each of which is subjected to soft failure due to the failure tolerance design. The non-periodic inspection scheme give a guaranteed level of reliability throughout the life of the system and at the same time reduces the inspection cost.

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ADVANCED COMPUTATIONAL METHODS FOR OPTIMIZATION OF NON-PERIODIC INSPECTION INTERVALS FOR AGING INFRASTRUCTURE

NIHON NO SHORAIWO KANGAERU KAI

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Contents

This is the report on the basic research for AOARD entitled "Advanced computational methods for optimization of non-periodic inspection intervals for aging infrastructure".

1. Introduction

There is a demand of performing maintenance of aging infrastructure more efficiently and effectively in many industries. Condition-based maintenance policy has been applied, significantly decreased the cost of repairing/replacing of damaged/failure parts. Some optimized inspection schemes are proposed considering the total life cost including inspection costs, repair costs, and the penalty costs of system failure. However, for a safety-critical system in which system failure is not allowed, the optimization of the inspection scheme is due to the decreasing of inspection cost while guaranteeing a certain level of reliability of whole system.

This report proposes an approach for optimization of non-periodic inspection scheme on a finite time horizon for a multi-component safety-critical system. The system consists of several components, each of which is subjected to soft failure due to the failure tolerance design. The non-periodic inspection scheme gives a guaranteed level of reliability throughout the life of the system and at the same time reduces the inspection cost.

The approach presented in this report is an improvement of a former article [1] in which a Bayesian method for non-periodic inspection of aircraft structures is introduced. The Bayesian method is improved by applying conditional probability into the simulations and gives more convenient in application. By Bayesian updating, the uncertain parameters can be estimated appropriately and reasonable inspection interval is scheduled.

Statistical analysis of this approach are performed. Results show that this advanced approach can reduce inspection cost and at the same time maintain the reliability level. The reliability of the estimated reliability is discussed and the presented approach has high reliability for practical usage.

An application example for turbine engine components is shown in this report. The presented approach still can optimized the non-periodic inspection interval when the parameter of crack propagation function is uncertain, while the normal Retirement–For-Cause (RFC) procedure meets difficulties. Furthermore, the probability of detection (POD) and the random failure can be considered by introducing fixed parameters, which is an improvement comparing with RFC.

The contents of this report include:

1) Non-periodic inspection scheme using probabilistic method
2) Bayesian method for uncertain parameters
3) Statistical analysis of the approach presented in this report
4) Improvement of Bayesian method by applying conditional probability
5) Application example for turbine engine components

2. Non-periodic inspection scheme using probabilistic method

Fatigue is one of the most important problems of aging infrastructure subjected to random dynamic loads. Fatigue damage is considered to initiate in structural element and continues by crack propagation, resulting in strength degradation. Periodic inspections are common practice in order to maintain their reliability above a desired level. However, considering the initiation and propagation of fatigue cracks as time goes by, it is obvious reasonable to perform non-periodic inspections instead.

2.1. Basic assumption and equations

The whole or part of the aging infrastructure as well as aircraft is refereed as system hereafter. A system is considered to consist of a specific number of elements. An element is defined so that it possesses only one fatigue-critical location. Throughout this report, time is measured in number of total accumulated cycles.

2.1.1. Crack, failure and inspection

All elements are inspected at the initiation of service and at the time of each scheduled inspection. Cracks and failures (soft failure which do not cause system un-functioning) can only be detected during inspection. The following assumptions are made:

- The probability of detecting of a crack is a function of crack length, which is shown later.
- The probability of detecting element failure is equal to unity.
- All cracks and failures are repaired or replaced when detected.
- After repair/replacement, element regains its initial strength (same as a new one).
- No stress redistribution is considered after the occurrence of crack and failure.

2.1.2. Fatigue crack initiation

The time to crack initiation (TTCI), denoted by t and measured in number of cycles, is assumed to be a random variable with density function following the Weibull distribution:

$$f_c(t|\beta) = \frac{\alpha}{\beta} \cdot \left(\frac{t}{\beta}\right)^{\alpha-1} \cdot exp\left[-\left(\frac{t}{\beta}\right)^{\alpha}\right] \qquad t > 0. \qquad (2\text{-}1)$$

Additional uncertainty is introduced in the TTCI by a scale parameter β which is considered to be a random variable. Therefore, eq. (2-1) indicates a Weibull density function conditional to a given value of β. This is the first parameter which is considered uncertain in this study. The shape parameter α is assumed to be deterministic for the sake of simplicity. The distribution function of the TTCI is expressed by:

$$F_c(t|\beta) = 1 - exp\left[-\left(\frac{t}{\beta}\right)^{\alpha}\right] \qquad t > 0. \tag{2-2}$$

2.1.3. Fatigue crack propagation

Fracture mechanics theory is used to determine the length of a propagating crack under random stress. For the purpose of this study, it is assumed that a crack grows according to the following law according to article [2]:

$$\frac{\mathrm{d}a}{\mathrm{d}t} = c \cdot a^{b/2}\,, \tag{2-3}$$

where a is the crack length, c and b are constants. Integrating eq. (2-3) from the initial crack length a_0 at time of crack initiation t_c, up to the current crack length a at time t, the following expression is obtained:

$$a(t - t_c|c) = \left[-b'c(t - t_c) + a_0{}^{-b'}\right]^{-1/b'} \quad \text{where } b' = \frac{b-2}{2}\,. \tag{2-4}$$

Uncertainty in fatigue crack propagation is introduced by parameter c which is considered to be a random variable. Therefore, the crack length indicated by eq. (2-4) is conditional to a given value of c. This is the second parameter which is considered uncertain in this study.

2.1.4. Probability of detection

The probability of detecting an existing crack of length a during an inspection is given by:

$$D(a|d) = 1 - exp\left[-\left(\frac{a - a_{min}}{d - a_{min}}\right)^{\theta}\right]. \tag{2-5}$$

Uncertainty in the probability of crack detection is introduced by parameter d which is considered to be a random variable and therefore the probability shown in eq. (2-5) is conditional to a given value of d. This is the third parameter which is considered uncertain in this study. Finally, a_{min} denotes the minimum detectable length and θ is a constant.

2.1.5. Failure rate and probability of safety

Failure of an element occurs when the random stress exceeds the strength of the element for the first time. An element can fail either before or after crack initiation. According to article [2], the failure rate before and after crack initiation at time instant t_c are given as followed.

Before crack initiation:

$$h(t) = exp(r) = h_0 \,. \tag{2-6}$$

After crack initiation:

$$h(t) = \frac{\alpha_r}{\beta_r} \cdot \left(\frac{t}{\beta_r}\right)^{\alpha_r - 1} + exp(r) \,. \tag{2-7}$$

For the sake of simplicity, parameter r, α_r, β_r, are assumed to be deterministic constants. Then, the probability of safety of an element before crack initiation during the service period from time instant T_l up to time instant t is denoted as U and given by:

$$U(t - T_l) = exp\left\{-\int_{T_l}^{t} h(\tau)\mathrm{d}\tau\right\} = exp\left\{-\int_{T_l}^{t} exp(r)\mathrm{d}\tau\right\} \tag{2-8}$$

or

$$U(t - T_l) = exp\{-(t - T_l) \cdot exp(r)\} \quad \text{for } t \le t_c \,, \tag{2-9}$$

where T_l is the time of service initiation for the element under consideration. On the other hand, the probability of safety of an element after crack initiation during the service period from time instant of crack initiation t_c up to time instant t is denoted as V and given by:

$$V(t - t_c) = exp\left\{-\int_{t_c}^{t} h(\tau)\mathrm{d}\tau\right\} = exp\left\{-\int_{t_c}^{t} \left[\frac{\alpha_r}{\beta_r} \cdot \left(\frac{\tau}{\beta_r}\right)^{\alpha_r - 1} + exp(r)\right]\mathrm{d}\tau\right\} \tag{2-10}$$

or

$$V(t - t_c) = exp\left\{-\frac{1}{{\beta_r}^{\alpha_r}}(t^{\alpha_r} - {t_c}^{\alpha_r}) - (t - t_c) \cdot exp(r)\right\} \quad \text{for } t > t_c \,. \tag{2-11}$$

The probability of safety (sometimes refer as reliability) mentioned here is a conditional probability in the conditions of crack initiation happens or not. The true reliability of an element should be a sum of the probability of all conditions.

It should be pointed out that the functional forms used for fatigue crack initiation, crack propagation, probability of detection, failure rate and probability of safety are selected mainly to demonstrate the capabilities of the presented approach. It is a straightforward task to change any equations as well as parameters according to the objective, as shown in the application example in a later chapter.

2.2. Possible events and probabilities

At the time of j-th inspection performed at time T_j, all possible events and their probability must be considered in order to estimate the reliability of a certain element at any specific time.

2.2.1. Events and probabilities that a failure found

Event that the element is found to have failed at the time of the j-th inspection T_j (Equivalently, failure occurred during the time interval $[T_{j-1}, T_j]$) will be denoted as event $\{A: j, l\}$. This event consists of the following two mutually exclusive events:

(1) $E_{1,j}$ = event that the element failed before crack initiation, sometime between two inspections, during the time interval at $[T_{j-1}, T_j]$.

Event $E_{1,j}$ also consists of two sub-events $E_{1a,j}$ and $E_{1b,j}$. Event $E_{1a,j}$ is the sub-event of $E_{1,j}$ that no crack would have initiated in the element before T_j if failure did not occur sometime during the time interval $[T_{j-1}, T_j]$. The probability $P_{1a,j}$ of event $E_{1a,j}$ is given by:

$$P_{1\mathrm{a},j} = \{1 - F_c(T_j - T_l|\beta)\} \cdot \{U(T_{j-1} - T_l) - U(T_j - T_l)\}\,. \tag{2-12}$$

Event $E_{1b,j}$ is the sub-event of $E_{1,j}$ that a crack would have initiated in the element at time instant t ($T_{j-1} < t < T_j$) if failure did not occur during the time interval $[T_{j-1}, t]$. The probability $P_{1b,j}$ of event $E_{1b,j}$ is given by:

$$P_{1\mathrm{b},j} = \int_{T_{j-1}}^{T_j} f_c(t - T_l|\beta) \cdot \{U(T_{j-1} - T_l) - U(t - T_l)\}\mathrm{d}t\,. \tag{2-13}$$

(2) $E_{2,j}$ = event that the element failed after crack initiation, sometime between two inspections, during the time interval $[T_{j-1}, T_j]$.

Event $E_{2,j}$ also consists of two sub-events $E_{2a,j}$ and $E_{2b,j}$. Event $E_{2a,j}$ is the sub-event of $E_{2,j}$ that a crack initiated at time instant t in the time interval $[T_i, T_{i+1}]$ where $i = l, \ldots, j-2$. The crack was not detected during all subsequent inspections (from inspection T_{i+1} up to inspection T_{j-1} inclusive) and the element failed sometime during the time interval $[T_{j-1}, T_j]$. The probability $P_{2a,j}$ of event $E_{2a,j}$ is given by:

$$P_{2\mathrm{a},j} = \sum_{i=l}^{j-2} \left\{ \int_{T_i}^{T_{i+1}} f_c(t - T_l|\beta) \cdot U(t - T_l) \cdot \left[V\left(T_{j-1} - t\right) - V\left(T_j - t\right) \right] \right.$$

$$\left. \cdot \left[\prod_{k=i+1}^{j-1} \{1 - D(a(T_k - t|c)|d)\} \right] \mathrm{d}t \right\}. \qquad (2\text{-}14)$$

Event $E_{2\mathrm{b},\,j}$ is the sub-event of $E_{2,\,j}$ that a crack initiated in the element at time instant t in the time interval $[T_{j\text{-}1},\ T_j]$ and the element failed sometime during the time interval $[t,\ T_j]$. The probability $P_{2\mathrm{b},\,j}$ of event $E_{2\mathrm{b},\,j}$ is given by:

$$P_{2\mathrm{b},j} = \int_{T_{j-1}}^{T_j} f_c(t - T_l|\beta) \cdot U(t - T_l) \cdot \left\{1 - V\left(T_j - t\right)\right\} \mathrm{d}t\,. \qquad (2\text{-}15)$$

2.2.2. Events and probabilities that a crack found

Event that the element is found not to have failed at the time of the j-th inspection T_j and a crack of length between a_j and a_j + da_j is detected in the element will be denoted as event $\{B(a_j){:}\ j,\ l\}$. This event consists only one event and alternatively denoted by $E_{3,\,j}$.

Since a crack of length between a_j and a_j + da_j is detected at time of j-th inspection, the time instance t_c of initiation of the crack can be computed from

$$a_j = a\left(T_j - t_c|c\right) = \left[-b'c\left(T_j - t_c\right) + {a_0}^{-b'}\right]^{-1/b'} \qquad (2\text{-}16)$$

as:

$$t_c = T_j + \tfrac{1}{b'c}\left({a_j}^{-b'} - {a_0}^{-b'}\right). \qquad (2\text{-}17)$$

Differential dt_c can be calculated as:

$$\mathrm{d}t_c = \left|\frac{\mathrm{d}t_c}{\mathrm{d}a_j}\right| \mathrm{d}a_j = \frac{\mathrm{d}a_j}{c \cdot {a_j}^{b/2}}\,. \qquad (2\text{-}18)$$

The probability $p_{3,\,j}$ da_j of event $E_{3,\,j}$ is given by:

$$p_{3,j}\left(a_j\right)\mathrm{d}a_j = f_c(t_c - T_l|\beta)\mathrm{d}t_c \cdot U(t_c - T_l) \cdot V\left(T_j - t_c\right)$$

$$\cdot \left[\prod_{k=l+1}^{j-1} \{1 - \delta \cdot D(a(T_k - t_c|c)|d)\} \right] \cdot D\left(a_j|d\right). \qquad (2\text{-}19)$$

Substituting t_c of eq. (2-17) and dt_c of eq. (2-18) into eq. (2-19), the probability is expressed completely as a function of a_j where δ is given by:

$$\delta = \begin{cases} 1 & \text{for} \quad T_k > t_c \\ 0 & \text{for} \quad T_k < t_c \end{cases} . \tag{2-20}$$

2.2.3. Events and probabilities that nothing found

Event that the element is found not to have failed or cracked at the time of the j-th inspection T_j will be denoted as event $\{C: j, l\}$. This event consists of the following two mutually exclusive events:

(1) $E_{4,j}$ = event that the element did not fail during the time interval at $[T_{j-1}, T_j]$ and no crack exists in the element.

The probability $P_{4,j}$ of event $E_{4,j}$ is given by:

$$P_{4,j} = \left\{1 - F_c\left(T_j - T_l|\beta\right)\right\} \cdot U\left(T_j - T_l\right) . \tag{2-21}$$

(2) $E_{5,j}$ = event that the element did not fail during the time interval at $[T_{j-1}, T_j]$ but a crack exists in the element which is not detected during all subsequent inspections (from inspection T_{i+1} up to inspection T_j inclusive).

The probability $P_{5,j}$ of event $E_{5,j}$ is given by:

$$P_{5,j} = \sum_{i=l}^{j-1} \left\{ \int_{T_i}^{T_{i+1}} f_c(t - T_l|\beta) \cdot U(t - T_l) \cdot V\left(T_j - t\right) \cdot \left[\prod_{k=i+1}^{j} \{1 - D(a(T_k - t|c)|d)\} \right] \mathrm{d}t \right\}. \tag{2-22}$$

2.2.4. Conclusions of all events and probabilities

Finally, the probabilities of all events $\{A: j, l\}$, $\{B(a_j): j, l\}$ and $\{C: j, l\}$ are obtained as:

$$P\{A: j, l\} = P_{1\mathrm{a},j} + P_{1\mathrm{b},j} + P_{2\mathrm{a},j} + P_{2\mathrm{b},j} \tag{2-23}$$

$$P\left\{B\left(a_j\right): j, l\right\} = p_{3,j}\left(a_j\right)\mathrm{d}a_j \tag{2-24}$$

$$P\{C: j, l\} = P_{4,j} + P_{5,j}. \tag{2-25}$$

2.3. Reliability computation

The reliability of elements at time instant t^* during time interval $[T_j, T_{j+1}]$ can be calculated depending if the elements are repaired/replaced or not at the j-th inspection.

2.3.1. Reliability of elements repaired or replaced at the *j*-th inspection

An element is repaired or replaced at j-th inspection in the case of $\{A: j, l\}$ and $\{B(a_j): j, l\}$. All conditions of the element is reset so that the time of service initiation T_l is equal to T_j. As described in section 2.1.5., the reliability $R(t^*$: Rep.) of an element (in other words, the probability of element survival) is computed as the sum of the following two probabilities:

(1) probability that the element will survive during the time interval $[T_j, t^*]$ and no crack will initiate before t^*,

(2) probability that a crack will initiate in the element sometime during the time interval $[T_j, t^*]$, but the element will survive during the same time interval.

The reliability $R(t^*$: Rep.) is then calculated as:

$$R(t^*\colon \mathrm{Rep}) = \{1 - F_c(t^* - T_j|\beta)\} \cdot U(t^* - T_j) + \int_{T_j}^{t^*} f_c(t - T_j|\beta) \cdot U(t - T_j) \cdot V(t^* - t)\mathrm{d}t \, . \tag{2-26}$$

2.3.2. Reliability of elements not repaired nor replaced at the *j*-th inspection

An element is neither repaired nor replaced at j-th inspection in the case of $\{C: j, l\}$. Comparing with the case when repair/replacement happened, there is a possibility that a crack initiated before the j-th inspection and remained un-founded. The reliability $R(t^*$: No.) of an element is computed as the sum of the following three probabilities divided by the probability of event $\{C: j, l\}$(which is given by $P_{4,j} + P_{5,j}$):

(1) probability that the element will survive during the time interval $[T_l, t^*]$ and no crack will initiate before t^*,

(2) probability that a crack will initiate in the element sometime during the time interval $[T_j, t^*]$, but the element will survive during the time interval$[T_l, t^*]$,

(3) probability that a crack initiated in the element at some time instant t during the time interval $[T_i, T_{i+1}]$ where $i = l, \ldots, j$-1. This crack was not detected during all subsequent inspections (from inspection T_{i+1} up to inspection T_j inclusive) and the element will survive during time interval $[T_l, t^*]$.

The reliability $R(t^*$: No.) is then calculated as:

$$R(t^*\colon \text{No.}) = \frac{Z}{P_{4,j} + P_{5,j}}. \tag{2-27}$$

The expression for Z is given by:

$$\begin{aligned} Z = &\{1 - F_c(t^* - T_l|\beta)\} \cdot U(t^* - T_l) + \int_{T_j}^{t^*} f_c(t - T_l|\beta) \cdot U(t - T_l) \cdot V(t^* - t)\mathrm{d}t \\ &+ \sum_{i=l}^{j-1} \left\{ \int_{T_i}^{T_{i+1}} f_c(t - T_l|\beta) \cdot U(t - T_l) \cdot V(t^* - t) \right. \\ &\left. \cdot \left[\prod_{k=i+1}^{j} \{1 - D(a(T_k - t|c)|d)\} \right] \mathrm{d}t \right\}. \end{aligned} \tag{2-28}$$

2.3.3. Reliability with all parameters fixed

As mention earlier, parameters β, c and d are considered as possible sources of uncertainty. These uncertainties will be discussed and solved in the later chapter by applying Bayesian method. In this chapter, non-periodic inspection intervals with parameters fixed will be discussed first. In this case, the reliability of entire system of M elements at time instant t^* after the latest inspection T_i, is denoted by $\tilde{R}_M(t^*)$ and calculated as:

$$\tilde{R}_M(t^*) = \left[\prod_{m=1}^{M_1} R_m(t^*\colon \text{Rep.}) \right] \cdot \left[\prod_{m=1}^{M_2} R_m(t^*\colon \text{No.}) \right], \tag{2-29}$$

where M_1 = the number of elements repaired or replaced at j-th inspection, M_2 = the number of elements found intact at j-th inspection, and $M_1+M_2 = M$ which is the total number of entire system. R_m is the reliability of single element (denoted as m-th element) defined in eq. (2-26) and (2-27).

2.4. Numerical results and discussions

The system considered in this study is assumed to have 50 elements (M=50). Its service life is limited to 30,000 cycles and the minimum reliability level for entire system is set to 0.8 (R_{design}=0.8). In general, three uncertain parameters have been considered: β, c and d. However, discussions about non-periodic inspection with true values (deterministic values) are presented first in this chapter.

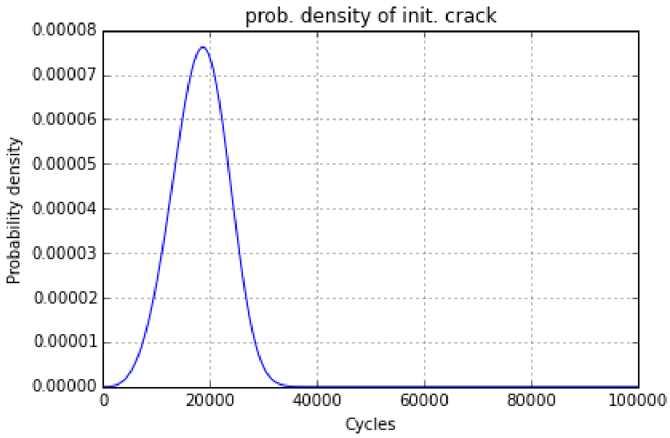
Table 2-1 Values of parameters in numerical simulation

Item		True values	Uncertain range
General	Design life limitation	30,000 cycles	
	Minimum level of R_{design}	0.8	
	Number of element M	50	
Initiation:	Parameter α	4.0	
Eq. (2-1)	Parameter β	40,000 cycles	20,000 ~ 60,000
Propagation:	Parameter b	2.96	
Eq. (2-3)	Parameter c	$1.6*10^{-4}$mm$^{-0.48}$/cycle	$0.6*10^{-4}$ ~ $2.6*10^{-4}$
Eq. (2-4)	Initial crack length a_0	2.5mm	
Detectability:	Parameter a_{min}	2.5mm	
Eq. (2-5)	Parameter θ	1.4	
	Parameter d	40mm	20 ~ 60
Reliability:	Parameter r	-14.5	
Eq. (2-6)	Parameter α_r	3.7	
Eq. (2-7)	Parameter β_r	8,000 cycles	

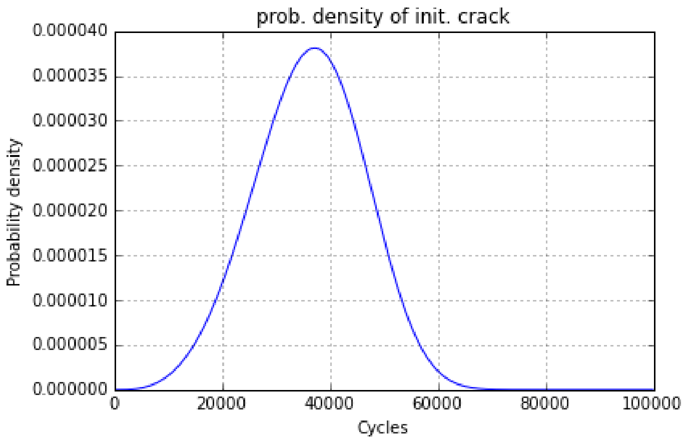
Table 2-1 shows the values of all parameters (uncertain and deterministic) involved in the problem. Note that the three uncertain parameters β, c and d are given true (deterministic) values along with their ranges (indicating uncertainty).

2.4.1. Survey of parameters

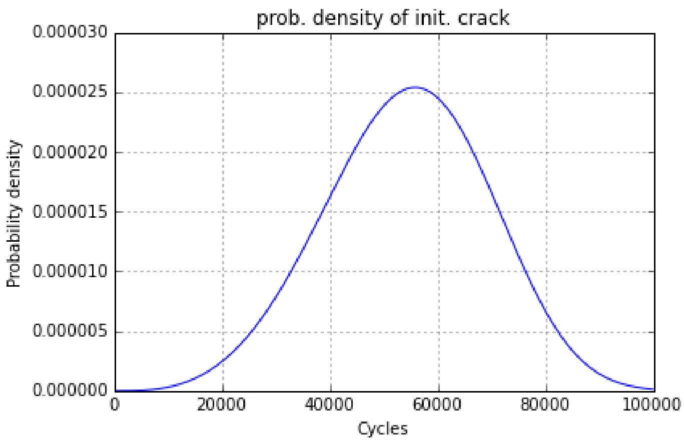
First, parameters β correspond to the initiation of a fatigue crack is investigated. Fig. 2-1 shows the probability density of crack initiation with different parameter β. Initiation of fatigue cracks happens earlier when parameter β is smaller. The peak values of probability density shown in figures are nearly the same as the parameter β itself. The integrated values along the time axis, which is the probability function of crack initiation are shown in Fig. 2-2. The probabilities of crack initiation in a single element within the service life limitation (30,000 cycles) are 0.994, 0.271 and 0.060 respectively. Considering the entire system consists of 50 elements, the average number of fatigue cracks initiated in the whole service life of this system is 49.7, 13.5 and 3.0. These number will greatly affect the reliabilities of the system and inspection scheme should be different when parameters β changes. A wrong prediction of this parameter will result in a totally wrong inspection scheme.



(a) Parameter β = 20,000

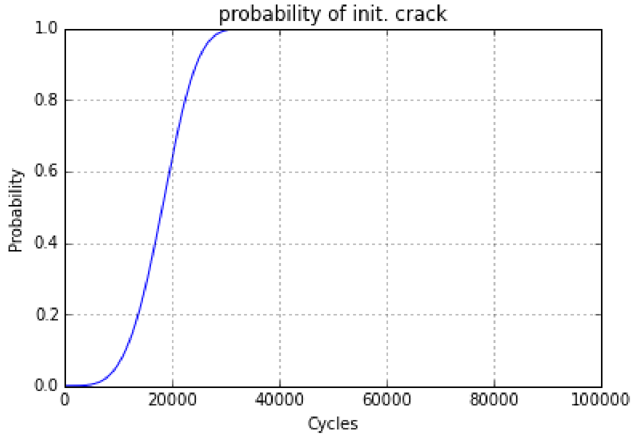


(b) Parameter β = 40,000

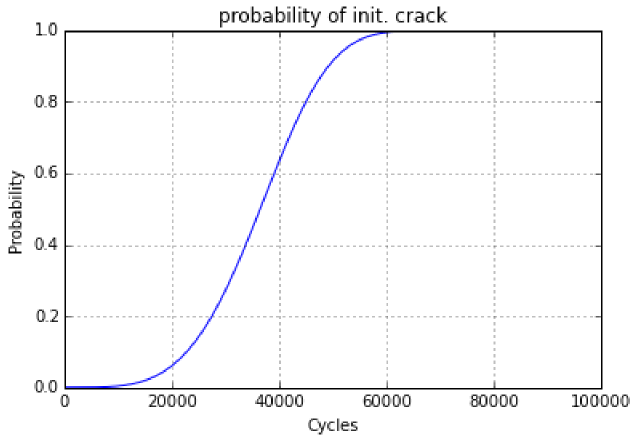


(c) Parameter β = 60,000

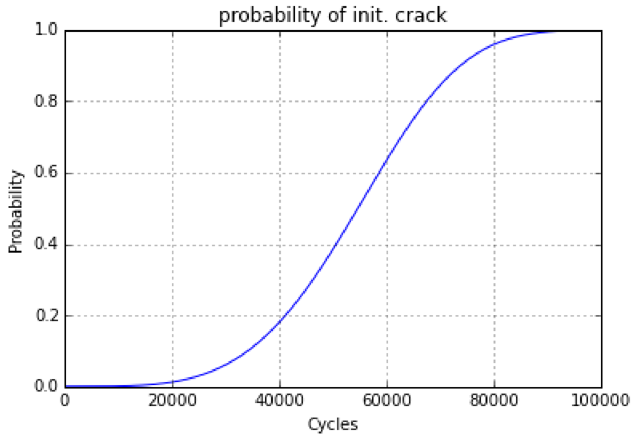
Fig. 2-1 Probability density of crack initiation



(a) Parameter β = 20,000



(b) Parameter β = 40,000



(c) Parameter β = 60,000

Fig. 2-2 Probability of crack initiation

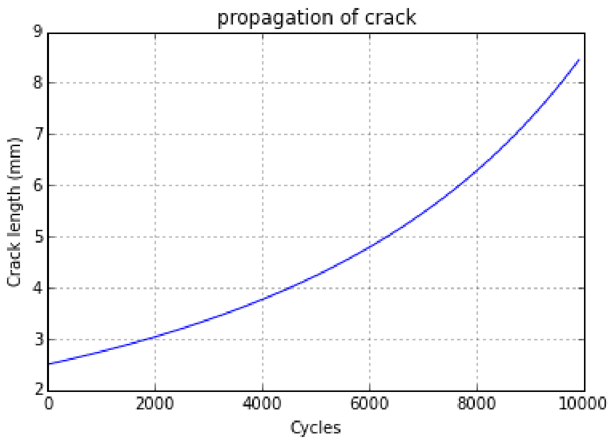
Parameters c correspond to the propagation of a fatigue crack affects the growth speed of crack length. Fig. 2-3 shows the crack length as a function of time since the initiation of a fatigue crack with different parameter c. It is obvious that propagation speed of a fatigue crack changes when parameter c changes. However, the crack length is not directly connected to the reliability of an element. Depending on the parameter c, events happen at the time of inspection will be different because the cracks are longer and are more likely be found during inspection. However, it is clear that the reliability is less sensitive to c than to β.

Parameters d correspond to the probability of detection of a crack affects the inspection results at each scheduled inspection time. Fig. 2-4 shows probability of detection as a function of crack length with different parameter d. Parameters d have nothing to do with the reliability of an element nor the crack size. It is clear that the parameters d is the least important factor to the reliability of an element as well as to the inspection scheme.

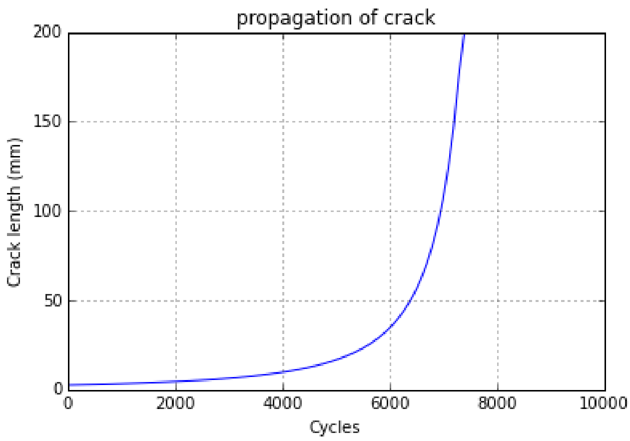
2.4.2. Periodic inspection scheme

Because of the lack of actual data, numerical simulations are performed in order to get a virtual system that follow all the assumptions and equations describe at the beginning of this chapter. All parameters in the equations are assumed to be deterministic as shown in Table 2-1. Periodic inspection scheme with an inspection interval of 2500, 2000, 1500, 1000 and 750 cycles are applied. The reliabilities during the service life with different periodic inspection intervals are estimated using probabilistic analysis method.

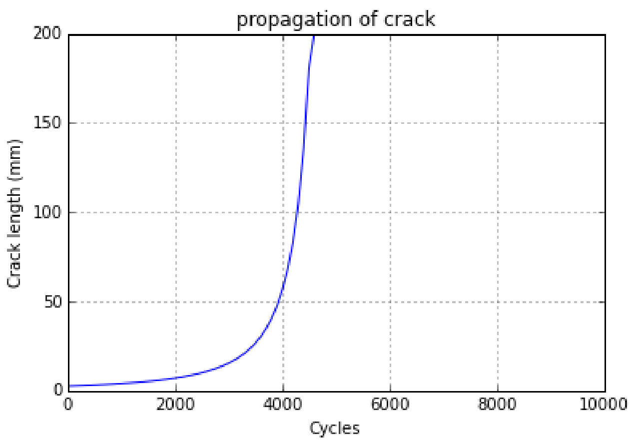
Reliabilities for the periodic inspections with different intervals are shown in Fig. 2-5 ~ Fig. 2-9, where figure (a) is the reliability of a single element and (b) is the reliability of entire system with 50 elements. For the reliability of a single element shown in figure (a), reliabilities of all elements change in the same way at the very beginning because basically all elements are in the same conditions. However, a few of the elements are repaired or replaced due to crack or failure and the curve of reliability separates into two. Although there are some elements are repaired or replaced and the reliability of these elements are restored, the reliability of entire system goes downward faster because of the aging of the un-replaced elements. The reliabilities of entire system go beneath the designed minimum level eventually at the time of 20000, 22000, 24000, 27000 cycles when using periodic inspection interval of 2500, 2000, 1500 and 1000 cycles. Only the inspection interval of 750 cycles keeps the reliability above the level of 0.8 but with many unnecessary inspections obviously.



(a) Parameter c = 0.6*10^{-4}

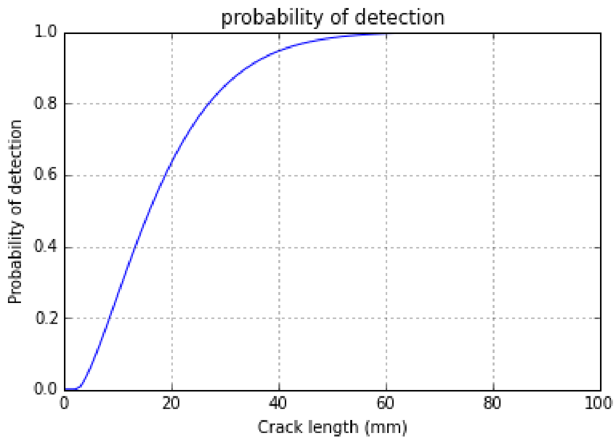


(b) Parameter c = 1.6*10^{-4}

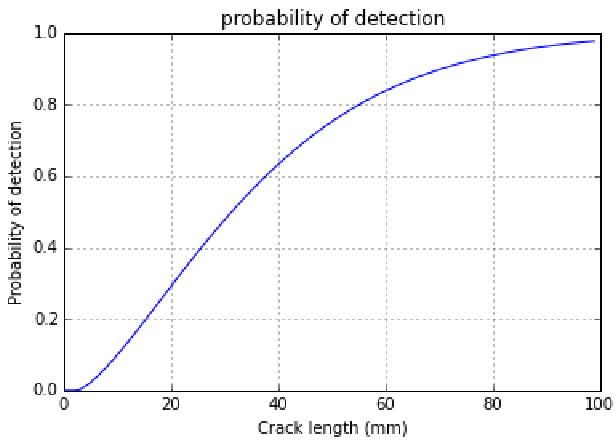


(c) Parameter c = 2.6*10^{-4}

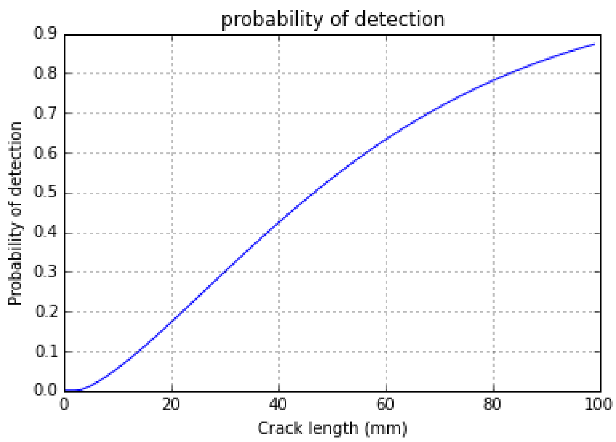
Fig. 2-3 Crack length as a function of time



(a) Parameter *d* = 20

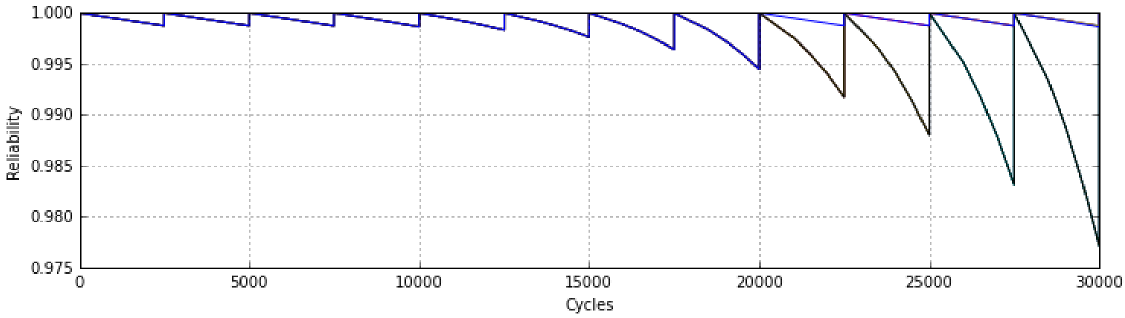


(b) Parameter *d* = 40

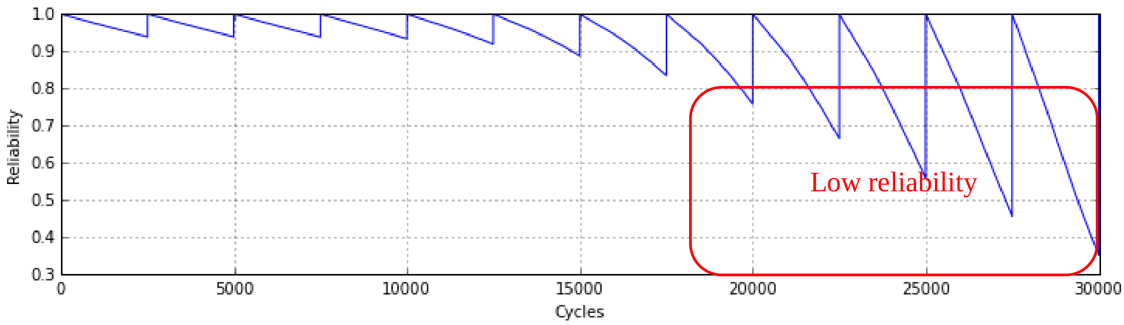


(c) Parameter *d* = 60

Fig. 2-4 Probability of detection as a function of crack length

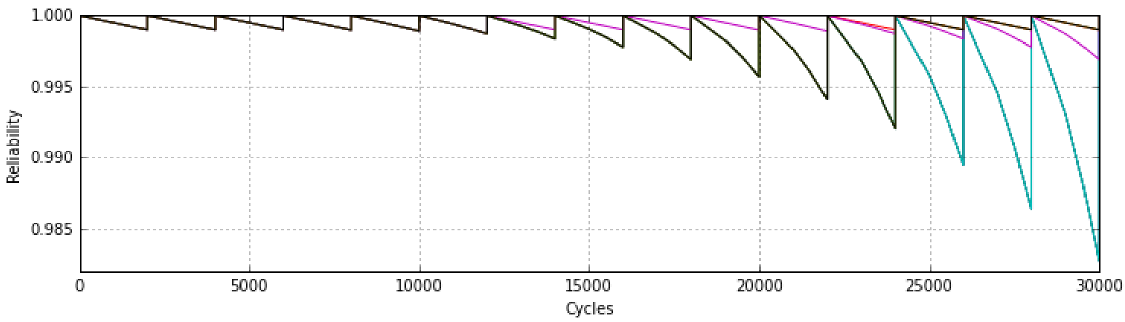


(a) Reliability of a single element

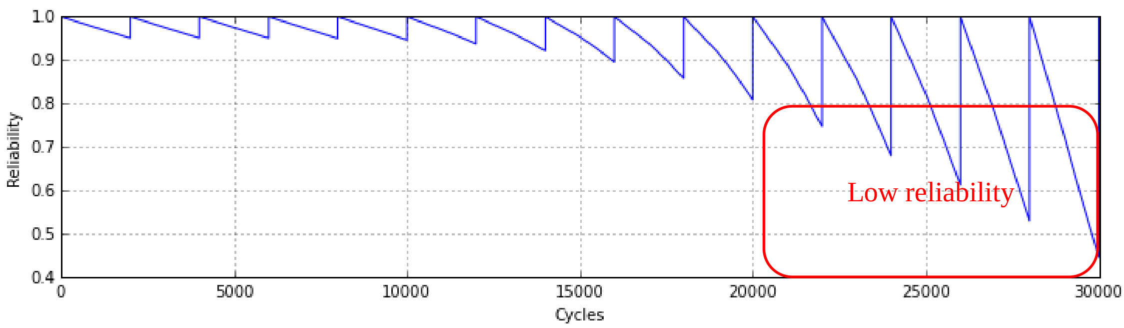


(b) Reliability of entire system with 50 elements

Fig. 2-5 Reliability for the periodic inspection with an interval of 2500 cycles

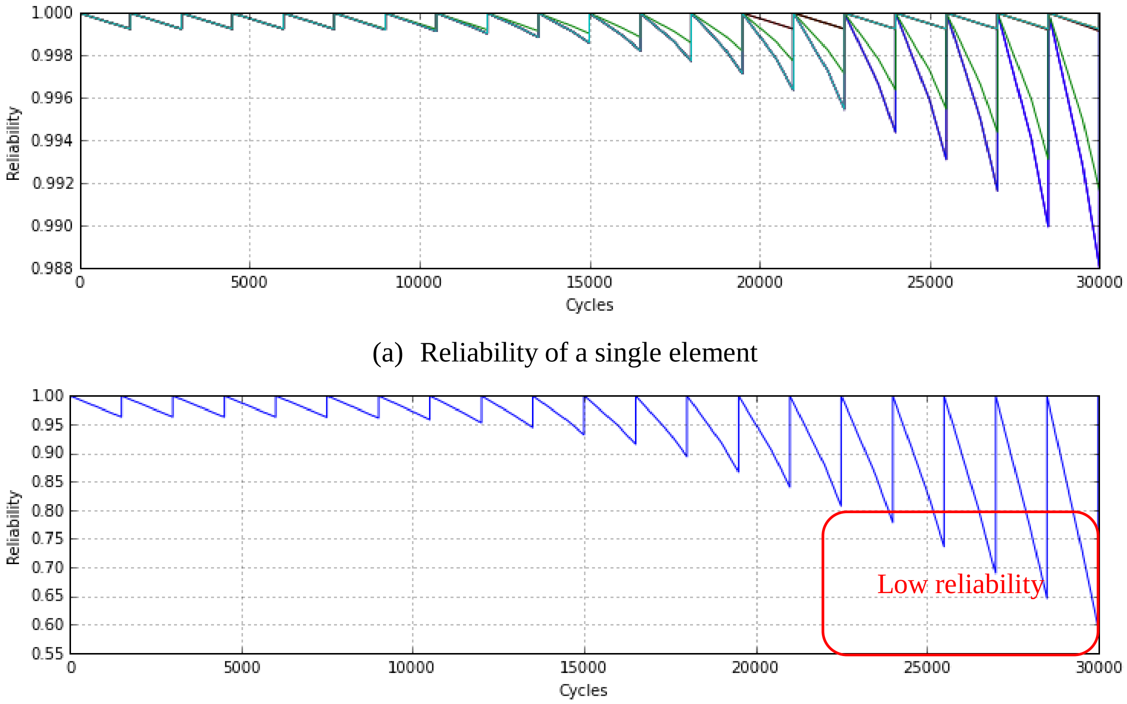


(a) Reliability of a single element



(b) Reliability of entire system with 50 elements

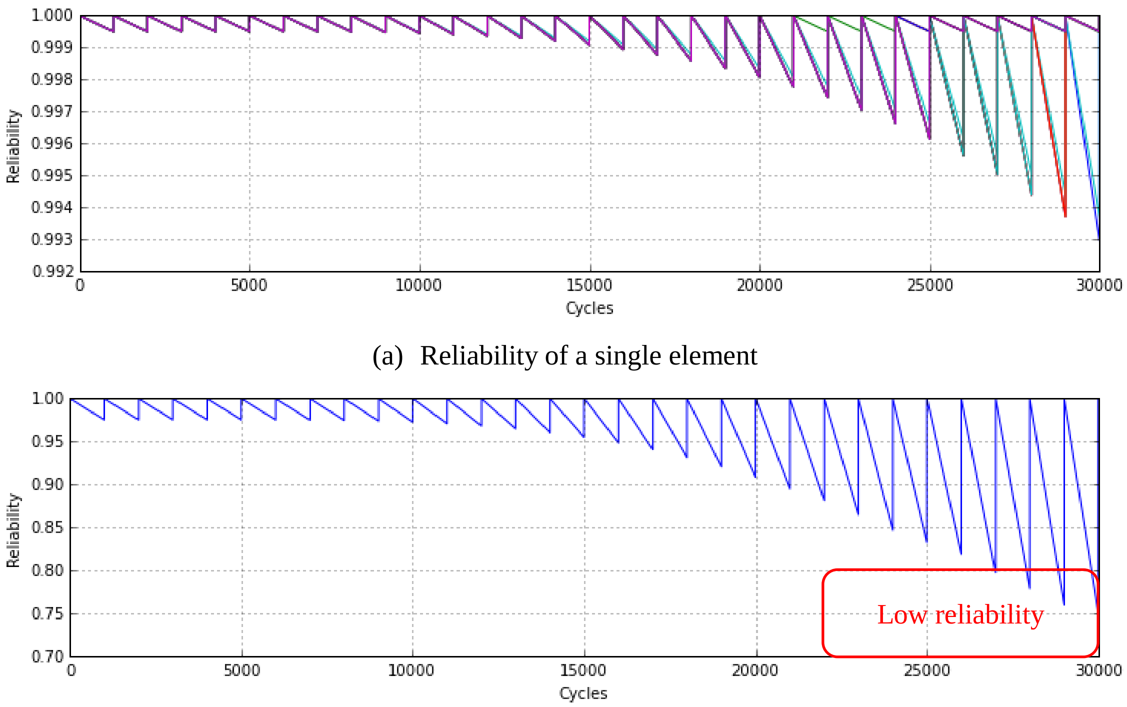
Fig. 2-6 Reliability for the periodic inspection with an interval of 2000 cycles



(a) Reliability of a single element

(b) Reliability of entire system with 50 elements

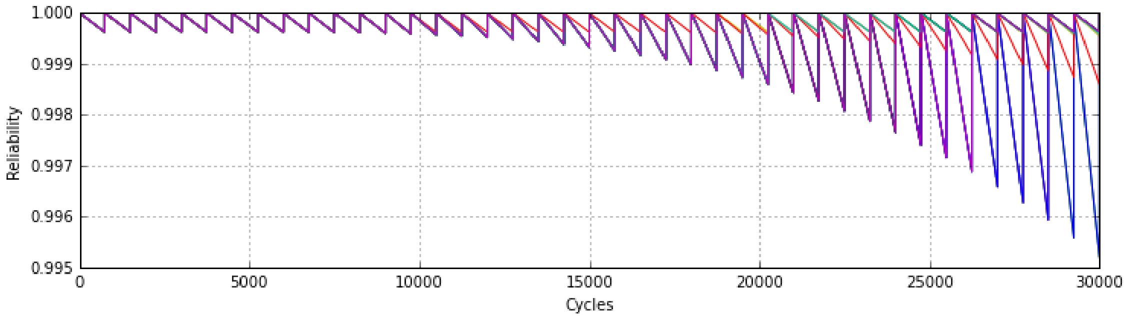
Fig. 2-7 Reliability for the periodic inspection with an interval of 1500 cycles



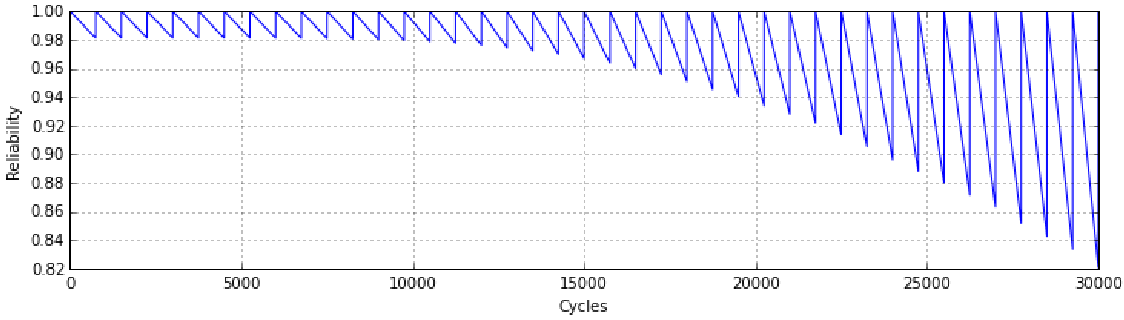
(a) Reliability of a single element

(b) Reliability of entire system with 50 elements

Fig. 2-8 Reliability for the periodic inspection with an interval of 1000 cycles



(a) Reliability of a single element



(b) Reliability of entire system with 50 elements

Fig. 2-9 Reliability for the periodic inspection with an interval of 750 cycles

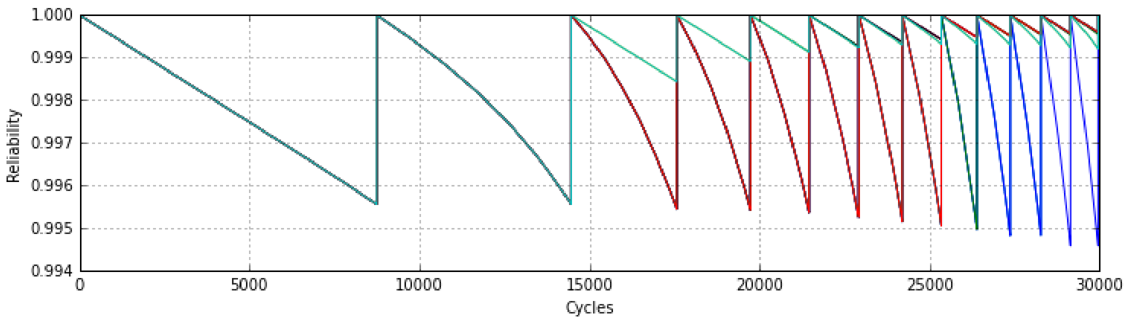
2.4.3. Non-periodic inspection scheme

Numerical simulations are performed in order to get a virtual system the same as last section. It should be pointed out that this virtual system follows equations with random distribution, that is to say, the output of each simulation is not exactly the same. Non-periodic inspections intervals are predicted by computing the reliability step by step and maintain the reliability above the minimum level 0.8. Once the estimated reliability of next step is going to be lower than expected level, an inspection is performed and the reliability regain 1.0.

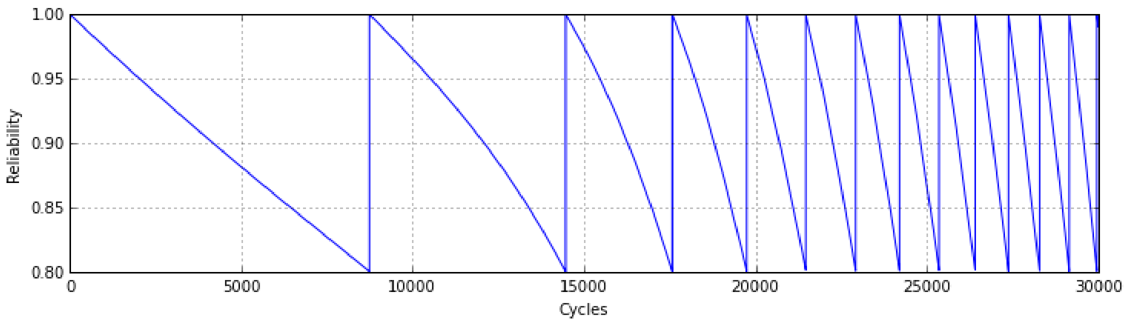
Reliabilities for the non-periodic inspections are shown in Fig. 2-10, where figure (a) is the reliability of a single element and (b) is the reliability of entire system with 50 elements. Same characteristics of the reliability as the periodic inspection scheme can be found, such as that the reliability of entire system goes downward faster while time goes by. By applying the non-periodic inspection scheme, the reliabilities of entire system maintains the required minimum level of 0.8. The inspection schedule are shown in Table 2-2. The interval between inspections is very long at the very beginning and subsequently becomes shorter as a function of time. It is a logical consequence for the fact that as the system gets older, more frequent inspections are needed instead of using a uniform inspection interval. The reliability of a 13 time non-periodic inspections maintains nearly the same as the minimum reliability of a 40 time periodic inspections.

Table 2-2 Inspection schedule of non-periodic scheme for the case with true values

Inspection no.	Inspection time (cycles)	Inspection interval (cycles)
1	8740	8740
2	14460	5720
3	17570	3110
4	19740	2170
5	21460	1720
6	22920	1460
7	24200	1280
8	25350	1150
9	26400	1050
10	27380	980
11	28280	900
12	29150	870
13	29960	810



(a) Reliability of a single element



(b) Reliability of entire system with 50 elements

Fig. 2-10 Reliability for the non-periodic inspection

2.4.4. Comparison of non-periodic and periodic inspection

Periodic and non-periodic inspection schemes using probabilistic analysis method are discussed in this chapter. The time intervals, number of inspections and minimum reliability for entire system of each inspection scheme are summarized in Table 2-3. Periodic inspection schemes are easy to apply and suitable for the cases that all fragile parts are replaced during overhaul. However, when condition based maintenance are more and more adopted, a non-periodic inspection scheme would be a more straightforward solution. According to the numerical results, a non-periodic inspection scheme which consists of 13 times inspections maintains the minimum reliability over 0.80, while a periodic 15 times inspections scheme has a minimum reliability of only 0.46. On the other hand, at least 40 times periodic inspections are needed to get the minimum reliability of 0.81.

The reliabilities shown here are computed by a probabilistic analysis method, using some assumptions and equations to represent the initiation of a fatigue crack, propagation of a crack and the probability of detection of a crack. The problem of the uncertainty of some parameters still exists, but no matter how these parameters are, non-periodic inspection is undeniable superior to periodic inspection.

Table 2-3 Comparison of non-periodic and periodic inspection

Inspection type	Time interval	Number of Inspections	Minimum reliability
1. Periodic	2500 cycles	12	0.35
2. Periodic	2000 cycles	15	0.46
3. Periodic	1500 cycles	20	0.59
4. Periodic	1000 cycles	30	0.73
5. Periodic	750 cycles	40	0.81
6. Non-periodic	810 ~ 8740 cycles	13	0.80

2.5. Summary

Main contents and results obtained in this chapter are summarized as follows:

(1) A probabilistic analysis method is introduced to estimate the reliability of a multi-elements system in order to optimize the inspection scheme.

(2) Periodic and non-periodic inspection schemes are discussed.

(3) Non-periodic inspection scheme are optimized to maintain a required minimum reliability. It is obviously superior to periodic inspection because of higher reliability and less inspections.

3. Bayesian method for uncertain parameters

It is assumed that the fatigue crack considered in the elements followed the equations with some parameters given. However, these parameters are not easily obtained and change depending on environment conditions. In order to solve the problem of these uncertainties, Bayesian method is applied, thus these parameters are modified according to the information from inspection results.

3.1. Bayesian analysis

3.1.1. Prior joint density function

As mentioned earlier, parameters β, c and d are considered as possible sources of uncertainty. Initially, it is assumed that β, c and d are jointly and uniformly distributed according to the following prior density function:

$$f^0(\beta, c, d) = \frac{1}{(\beta_{max} - \beta_{min})(c_{max} - c_{min})(d_{max} - d_{min})} = \text{constant} = f^0, \qquad (3\text{-}1)$$

where

$$\beta_{min} \le \beta \le \beta_{max}; \;\; c_{min} \le c \le c_{max}; \;\; d_{min} \le d \le d_{max} \,. \qquad (3\text{-}2)$$

3.1.2. Likelihood function

The likelihood function LF_j for the entire system as a result of the j-th inspection is calculated as:

$$LF_j = \prod_{m=1}^{M} LF_j^{(m)} \,, \qquad (3\text{-}3)$$

where $LF_j^{(m)}$ is the likelihood function for element m resulting from the j-th inspection and M is the total number of elements in the system. For a specified element m, consider that replacement or repair occurred at inspections $T_{l_1}, T_{l_2}, \dots, T_{l_r},$ where r indicates the number of times the element has been repaired or replaced before the j-th inspection. Consequently,

$$l_1 < l_2 < \cdots < l_r < j \,. \qquad (3\text{-}4)$$

It is pointed out that $l_1, l_2, \dots\ l_r$ are all known at the time of the j-th inspection since the entire inspection history of each element is considered to be known. The prior density function eq. (3-1) is supposed to be the possible distribution at the time of the initiation of service and the likelihood function of an element needs to consider from service start. This may be not convenient when not all the historical information of inspections are available and someone wants to start the simulation in the middle of a service life. Bayesian analysis method using conditional probability will be discussed in a later chapter by which can solve this problem.

The likelihood function for element *m* resulting from the *j*-th inspection is given by:

$$LF_j^{(m)} = P_m\{X: j, l_r\} \cdot \prod_{k=1}^{r} P_m\{Y: l_k, l_{k-1}\}\,. \tag{3-5}$$

In eq. (3-5), *X* stands for either even *A*, *B* or *C* depending on the result of the *j*-th inspection for element *m*, *Y* stands for either *A* or *B* depending on the result of the l_k-th inspection for element *m*. The probability $P_m\{*\}$ refers to eq. (2-23), (2-24) and (2-25). Finally, for the case where element *m* is found intact at all inspections prior to the *j*-th, the $r = 0$ and l_0 denotes the time of initiation of service. The product appearing at the right-hand-side of eq. (3-5) is set equal to unit. It is very important to note that the likelihood function defined in eq. (3-5) is obviously conditional to given values of *β*, *c* and *d*.

3.1.3. Posterior joint density function

The posterior joint density function of parameters *β*, *c* and *d* after the *j*-th inspection is given by:

$$f^j(\beta, c, d) = \frac{LF_j \cdot f^0}{\int_{\beta_{min}}^{\beta_{max}} \int_{c_{min}}^{c_{max}} \int_{d_{min}}^{d_{max}} (\text{Numerator}) \mathrm{d}\beta \mathrm{d}c \mathrm{d}d}\,. \tag{3-6}$$

It is clear that likelihood function LF_j is conditional to given values of *β*, *c* and *d*.

3.2. Calculations of reliability and next inspection interval

The reliability of the entire system considering a distribute function of uncertain parameters *β*, *c* and *d* is calculated by an integral over whole variable space as following:

$$\overline{\tilde{R}_M}(t^*) = \int_{\beta_{min}}^{\beta_{max}} \int_{c_{min}}^{c_{max}} \int_{d_{min}}^{d_{max}} \tilde{R}_M(t^* | \beta, c, d) \cdot f^j(\beta, c, d) \mathrm{d}\beta \mathrm{d}c \mathrm{d}d\,, \tag{3-7}$$

where

$$\tilde{R}_M(t^* | \beta, c, d) = \left[\prod_{m=1}^{M_1} R_m(t^*: \text{Rep.})\right] \cdot \left[\prod_{m=1}^{M_2} R_m(t^*: \text{No.})\right]. \tag{3-8}$$

Eq. (3-8) is a minor change of eq. (2-29) which computing reliabilities at all discrete points over the domains of all uncertain parameters. The same discretization is applied to all uncertain parameters to compute likelihood function eq. (3-5) and posterior joint density function eq. (3-6). The integrations in Eq. (3-6) and (3-7) are also computed by numerical integral.

Assuming that the entire system must maintain its reliability above a pre-specified design level R_{design} throughout its service life, the reliability at time t^* after an inspection performed at T_j should satisfied that:

$$\overline{\tilde{R}_M}(t^*) \geq R_{\text{design}} . \tag{3-9}$$

The way to find the next inspection time is that to increase t^* by a time increment step by step and find the maximum value satisfied eq. (3-9).

The numerical simulation after inspection T_j (j =0, 1, …) are performed as follows:

(1) First stage, the virtual system starts to simulate the initiation of fatigue cracks, propagation of cracks and so on. True values of all parameters are used in this stage. These results are treated as real events happened in the virtual system.

(2) Second stage, estimate the reliability of entire system by probabilistic method, without knowing any information from first stage. The values of uncertain parameters are unknown, so the posterior joint density function of these parameters at time T_j are used in this stage. The maximum t^* which can satisfy eq. (3-9) is found and next inspection time T_{j+1} is set to this t^*.

(3) Virtual inspection is applied to the system at time T_{j+1}, according to the results from first stage.

(4) Inspection results are used to compute the likelihood function and posterior joint density function at time T_{j+1} .

(5) Go back to step 1, do numerical simulation for next inspection.

3.3. Numerical results of single uncertain parameter

As described above, three uncertain parameters have been considered: β, c and d. First, only single uncertain parameter is discussed.

3.3.1. Case 1: uncertain parameter β

The non-periodic inspection scheme with an uncertain parameter β is simulated by the approach presented in this study. The inspection schedule is shown in Table 3-1. The intervals between consequent inspections become smaller and smaller in most cases. However, because of the change of probability distribution function of parameter β, the intervals sometimes become longer.

The change of probability distribution function of parameter β is shown in Fig. 3-1. The prior distribution function is uniform and the posterior distribution function converge to the true value gradually. Without knowing the exact value of uncertain parameter β, it is possible to schedule the inspection scheme using Bayesian estimation method.

Reliabilities for the non-periodic inspections with an uncertain parameter β are shown in Fig. 3-2, and results show that the reliabilities of entire system maintains the required minimum level of 0.8 throughout the whole service life.

It should be pointed out that the results show here is only one example of this case. Statistical analysis are performed in chapter 4.

3.3.2. Case 2: uncertain parameter *c*

The non-periodic inspection scheme with an uncertain parameter *c* is simulated by the approach presented in this study. The inspection schedule is shown in Table 3-2. The intervals between consequent inspections become smaller and smaller in most cases. However, because of the change of probability distribution function of parameter *c*, the intervals sometimes become longer.

The change of probability distribution function of parameter *c* is shown in Fig. 3-3. The prior distribution function is uniform and the posterior distribution function converge to the true value gradually. The peak of density function of parameter *c* is only 0.1, which is lower than the case of parameter β. Although the value of uncertain parameter cannot concentrate to the true value better, it is still possible to schedule the inspection scheme using Bayesian estimation method.

Reliabilities for the non-periodic inspections with an uncertain parameter *c* are shown in Fig. 3-4, and results show that the reliabilities of entire system maintains the required minimum level of 0.8 throughout the whole service life. Statistical analysis are performed in chapter 4.

3.3.3. Case 3: uncertain parameter *d*

The non-periodic inspection scheme with an uncertain parameter *d* is simulated by the approach presented in this study. The inspection schedule is shown in Table 3-3. The intervals between consequent inspections become smaller and smaller in most cases.

The change of probability distribution function of parameter *d* is shown in Fig. 3-5. The prior distribution function is uniform and the posterior distribution function converge to the true value gradually. The peak of density function of parameter *d* is lowest in three cases. Although the value of uncertain parameter cannot concentrate to the true value better, the inspection intervals are still reasonable. Without knowing the exact value of uncertain parameter *d*, it is possible to schedule the inspection scheme using Bayesian estimation method.

Reliabilities for the non-periodic inspections with an uncertain parameter *d* are shown in Fig. 3-6, and results show that the reliabilities of entire system maintains the required minimum level of 0.8 throughout the whole service life. Statistical analysis are performed in chapter 4.

Table 3-1 Inspection schedule for the case 1 (uncertain parameter β)

Inspection no.	Inspection time (cycles)	Inspection interval (cycles)
1	8590	8590
2	13240	4650
3	16210	2970
4	18850	2640
5	21250	2400
6	23430	2180
7	25080	1650
8	25970	890
9	26690	720
10	27470	780
11	28270	800
12	29090	820
13	29780	690

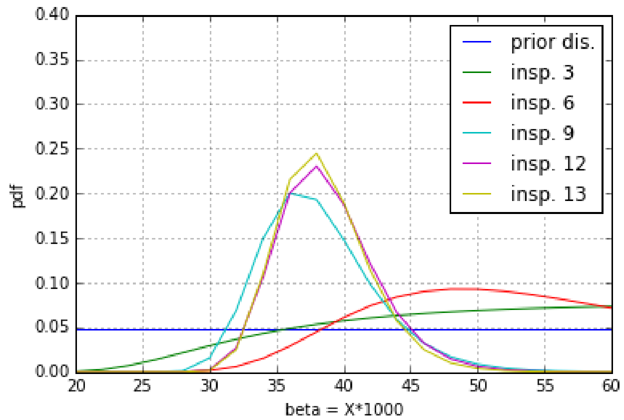


Fig. 3-1 Change of probability distribution function (uncertain parameter β)

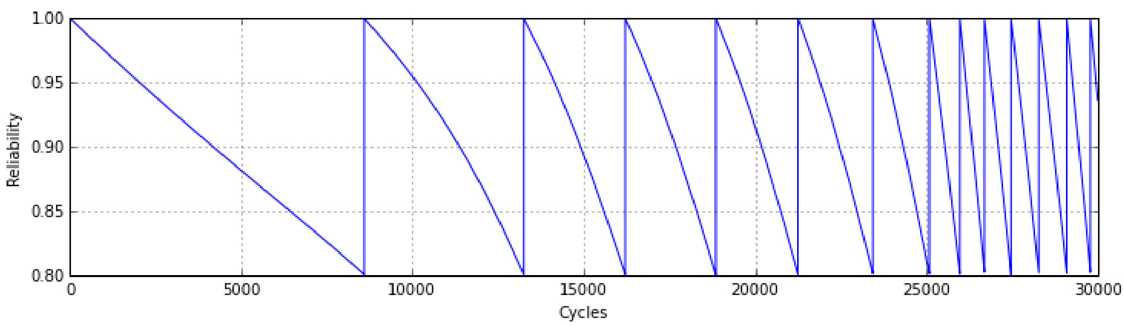


Fig. 3-2 Reliability for the case 1 (uncertain parameter β)

Table 3-2 Inspection schedule for the case 2 (uncertain parameter *c*)

Inspection no.	Inspection time (cycles)	Inspection interval (cycles)
1	8740	8740
2	14480	5740
3	17540	3060
4	19670	2130
5	21280	1610
6	23010	1730
7	24450	1440
8	25680	1230
9	26760	1080
10	27720	960
11	28590	870
12	29380	790

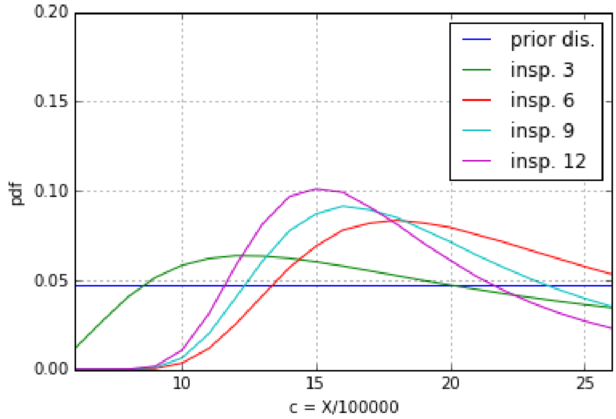


Fig. 3-3 Change of probability distribution function (uncertain parameter *c*)

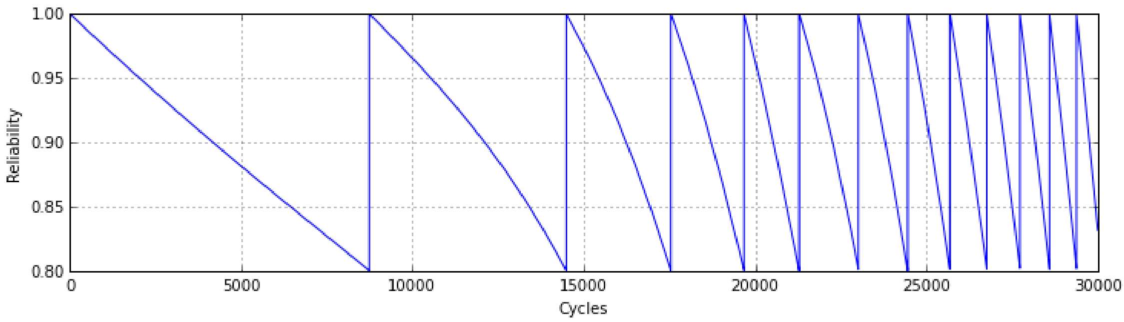


Fig. 3-4 Reliability for the case 2 (uncertain parameter *c*)

Table 3-3 Inspection schedule for the case 3 (uncertain parameter *d*)

Inspection no.	Inspection time (cycles)	Inspection interval (cycles)
1	8740	8740
2	14470	5730
3	17550	3080
4	19710	2160
5	21490	1780
6	22970	1480
7	24210	1240
8	25310	1100
9	26300	990
10	27250	950
11	28170	920
12	29040	870
13	29850	810

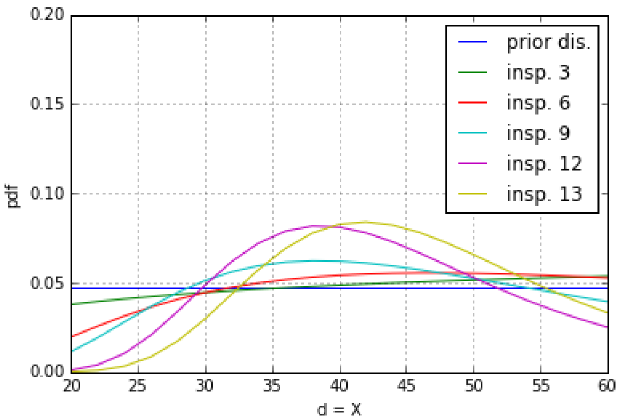


Fig. 3-5 Change of probability distribution function (uncertain parameter *d*)

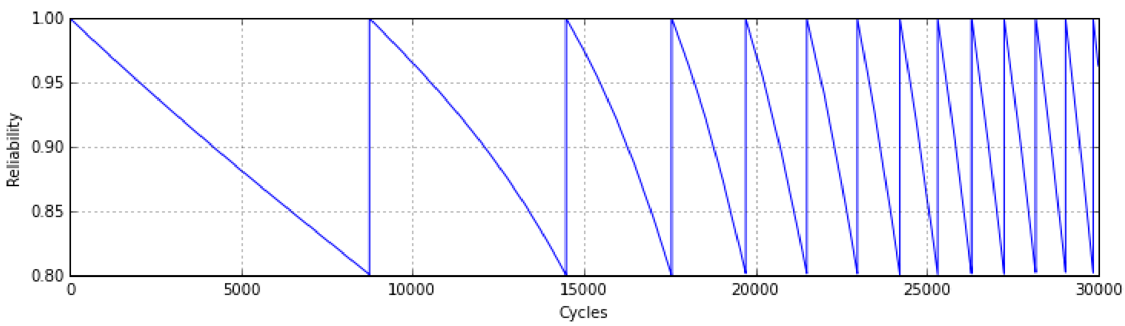


Fig. 3-6 Reliability for the case 3 (uncertain parameter *d*)

3.4. Numerical results of multiple uncertain parameters

Two of the three parameters β, c and d are supposed to be uncertain in this section. All combinations have been discussed.

3.4.1. Case 4: uncertain parameters β and c

The problem of non-periodic inspection scheme with two uncertain parameters β and c is solved by applying Bayesian estimation to the uncertain parameters. The prior distribution function is assumed to be a joint uniform distribution function. The posterior distribution function is computed as the product of the prior function and the likelihood function which derived from the information of inspection results. The change of probability distribution function of parameters β and c is shown in Fig. 3-7. The peak of the distribution function (36000 and 2.1E-4) is not always converge to the true value (40000 and 1.6E-4) because of local minimum problem and the lack of information. However, Bayesian estimation method gives an approximate estimation of the possible range of the parameters. Further discussions using statistical analysis are presented in chapter 4.

3.4.2. Case 5: uncertain parameters β and d

The problem of non-periodic inspection scheme with two uncertain parameters β and d is solved by applying Bayesian estimation to the uncertain parameters. The prior distribution function is assumed to be a joint uniform distribution function. The posterior distribution function is computed as the product of the prior function and the likelihood function which derived from the information of inspection results. The change of probability distribution function of parameters β and d is shown in Fig. 3-8. The peak of the distribution function (38000 and 38) is not always converge to the true value (40000 and 40) because of local minimum problem and the lack of information. However, Bayesian estimation method gives an approximate estimation of the possible range of the parameters. Further discussions using statistical analysis are presented in chapter 4.

3.4.3. Case 6: uncertain parameters c and d

The problem of non-periodic inspection scheme with two uncertain parameters c and d is solved by applying Bayesian estimation to the uncertain parameters. The prior distribution function is assumed to be a joint uniform distribution function. The posterior distribution function is computed as the product of the prior function and the likelihood function which derived from the information of inspection results. The change of probability distribution function of parameters *c* and *d* is shown in Fig. 3-9. The peak of the distribution function (1.7E-4 and 34) is not always converge to the true value (1.6E-4 and 40) because of local minimum problem and the lack of information. However, Bayesian estimation method gives an approximate estimation of the possible range of the parameters. Further discussions using statistical analysis are presented in chapter 4.

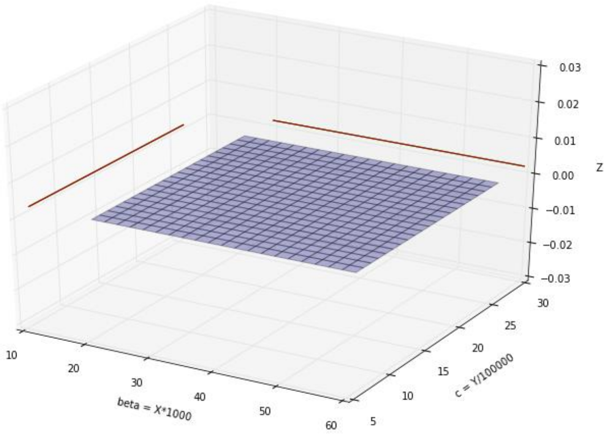
3.5. Summary

A Bayesian method is introduced to optimize the non-periodic inspection intervals with uncertain parameters. Contents and results are summarized as follows:

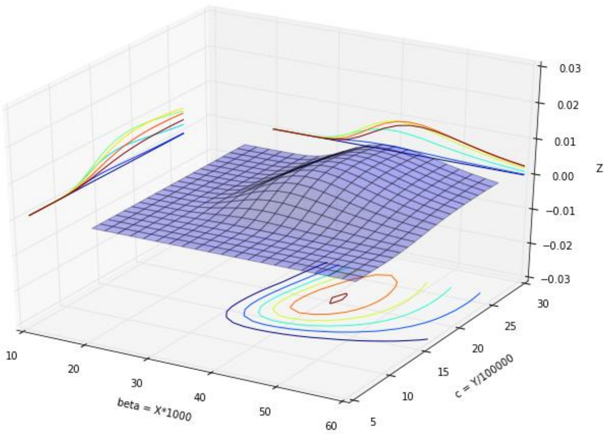
(1) Bayesian method is applied to deal with the uncertainty of parameters in non-periodic inspection scheme. Inspection intervals are estimated by the Bayesian approach and the estimated reliability of a multi-elements system maintains a pre-set level.

(2) The inspection schedule resulting from simulation of single uncertain parameter is presented. The intervals between consequent inspections become shorter as a function of time. The peak values of the posterior density functions are very close to the true values of the uncertain parameters.

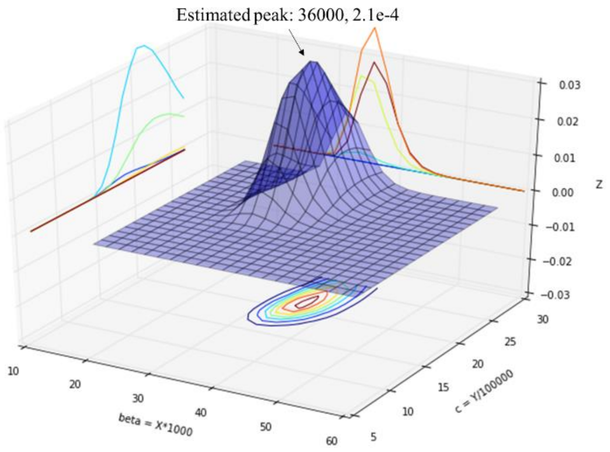
(3) The inspection schedule resulting from simulation of two uncertain parameters is presented. Because of the local minimum problem and the lack of information, the estimation of the true values of uncertain parameters consists large error sometimes.



(a) Prior probability density functions

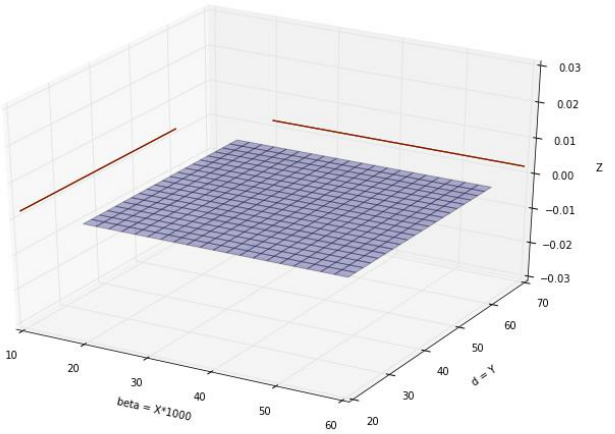


(b) Probability density functions after inspection 6

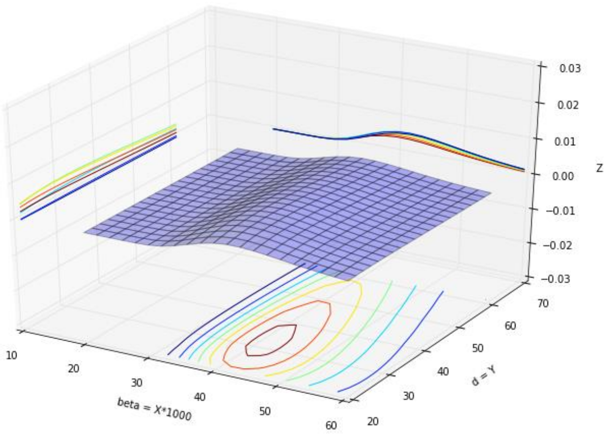


(c) Probability density functions after inspection 12

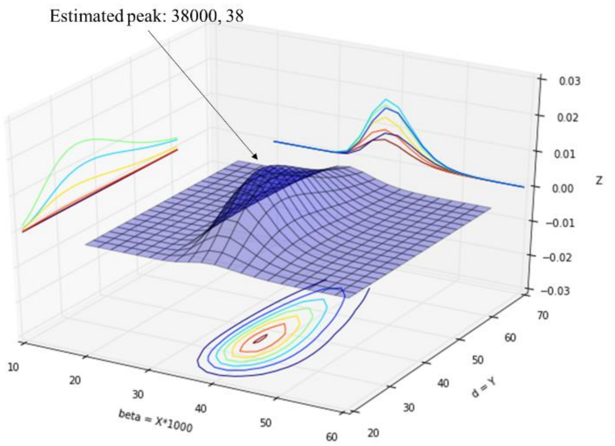
Fig. 3-7 Change of probability distribution function (uncertain parameters β and c)



(a) Prior probability density functions

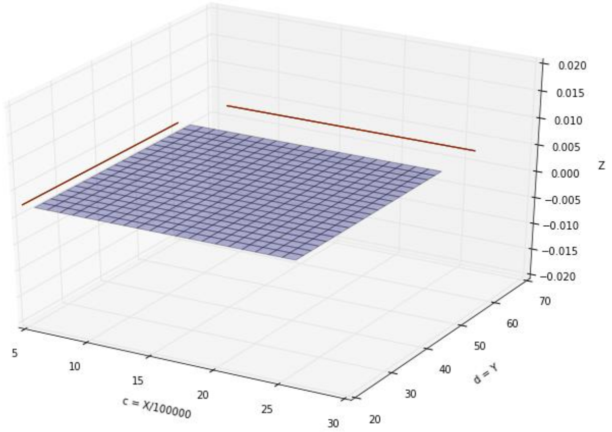


(b) Probability density functions after inspection 6

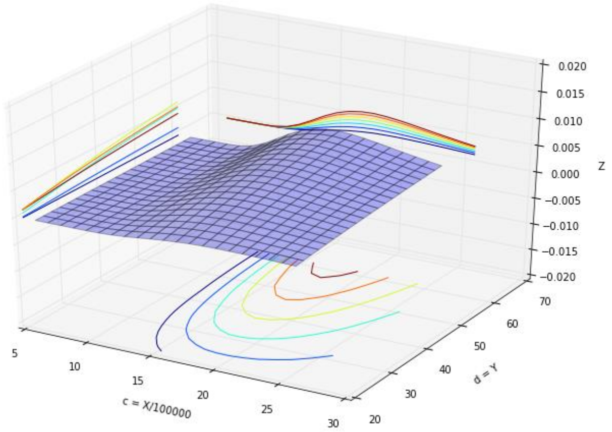


(c) Probability density functions after inspection 12

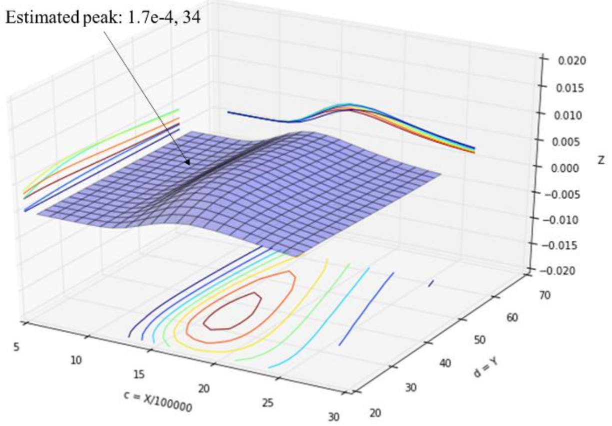
Fig. 3-8 Change of probability distribution function (uncertain parameters β and d)



(a) Prior probability density functions



(b) Probability density functions after inspection 6



(c) Probability density functions after inspection 11

Fig. 3-9 Change of probability distribution function (uncertain parameters *c* and *d*)

4. Statistical analysis

The approach presented in this study is based on probabilistic analysis. The initiation time of a fatigue crack is considered to be a random variable following the Weibull distribution. Furthermore, the parameters β, c and d which are involving in these equations may also be uncertain. The simulation results are not static but dynamic thus statistical analysis are necessary to evaluate the effect of this approach as well as the Bayesian estimation.

4.1. Cost reduction

The system consists of several components, each of which is subjected to soft failure. Soft failures of each component do not cause the system to stop functioning, but increase the system operating costs and are detected only if inspection is performed. The system's expected total cost associated with a given inspection scheme includes inspection costs, repair costs, and the penalty costs of a soft failure. The objective is to determine the optimal inspection scheme which minimizes system expected total cost.

4.1.1. Statistical analysis of non-periodic inspection with true value

In order to estimate the expected total cost of a given inspection scheme, statistical analysis of non-periodic inspection with true value are performed. The results of expected number of inspection times, number of failure elements and number of repair are shown in Table 4-1. Different sample number from 500 to 2000 are used and results show that 1000 or more samples are enough to obtain good statistical analysis results especially for the number of inspection times.

Table 4-1 Statistical results for different number of sample (non-periodic, true value)

Number of sample	Number of inspections		Number of failure		Number of repair	
	Average	SD	Average	SD	Average	SD
500	12.98	0.15	0.81	0.89	7.09	2.48
	12.98	0.18	0.90	0.97	7.30	2.58
	12.98	0.15	0.85	0.92	7.22	2.47
1000	12.98	0.16	0.85	0.93	7.19	2.53
	12.98	0.18	0.92	0.97	7.25	2.55
2000	12.98	0.17	0.88	0.95	7.22	2.54

From the results in Table 4-1, one can find that the standard deviation of number of inspection is small (less than 1%). But on the other hand, the standard deviation of number of failure elements and number of repair is very large. This is because that the initiation of fatigue crack as well as failure rate are randomly distributed in a large range.

4.1.2. Statistical analysis of different inspection schemes

Different inspection schemes are evaluated by statistical analysis including periodic inspection and non-periodic inspection. To decrease the inspection costs, a partial non-periodic inspection scheme is considered. This is easy to perform by ignoring inspection of the elements with higher reliability due to a recent replacement. Fig. 4-1 shows the reliability of single element during the service life. All reliabilities of single element are the same in the beginning. The curve separates when an element was replaced. For a full inspection scheme that inspect all elements, this curve will reunion at the time of next inspection. But for a partial inspection scheme introduced here, the element with higher reliability exempt from inspection in order to decrease the inspection costs. Both the reliability of a single element and that of entire system are kept above the required level in the inspection scheme.

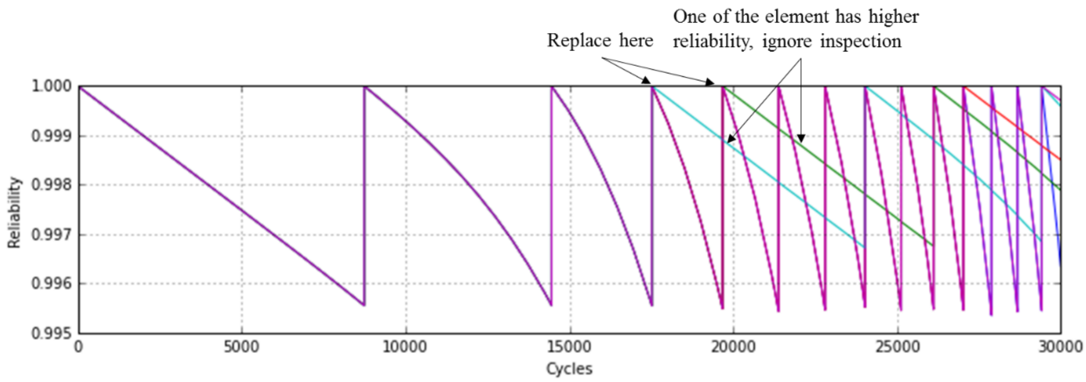


Fig. 4-1 Reliability of single element for partial inspection scheme (parameter with true value)

The results of expected number of inspection times, number of failure elements and number of repair for different inspection scheme are shown in Table 4-2. For periodical inspection scheme, the number of failure decreases a little bit when inspection interval decreases, at the same time, the number of repair or replacement increase a little bit. The costs of these two parts cancel out each other and keep the sum of costs nearly a constant. In all cases shown in Table 4-2, the number of failure and repair are nearly the same. It is easy to conclude that the total expected costs mainly depend on the inspection cost only.

The inspection cost (number of inspections * number of elements) for some of the inspection schemes are shown in Fig. 4-2, and the minimum reliabilities are also compared in Fig. 4-3. The effect of decreasing inspection cost is great when applying non-periodic inspection. While at the same time, the minimum reliabilities of non-periodic inspection keep higher. The partial inspection scheme is possible to apply with nearly the same reliability with a full inspection scheme. The difference of inspection cost between these two schemes is only about 5%.

Table 4-2 Statistical results for different inspection scheme (1000 samples)

Inspection type	Number of inspections		Number of failure		Number of repair	
	Average	SD	Average	SD	Average	SD
Periodic 2500	12	0	0.99	0.97	7.39	2.50
Periodic 2000	15	0	0.91	0.95	7.32	2.53
Periodic 1500	20	0	0.86	0.95	7.55	2.56
Periodic 1000	30	0	0.74	0.87	7.60	2.61
Periodic 750	40	0	0.77	0.84	7.81	2.68
Non-periodic	12.98	0.17	0.88	0.95	7.22	2.54
Partial inspect	12.38	0.27	0.84	0.91	7.19	2.51

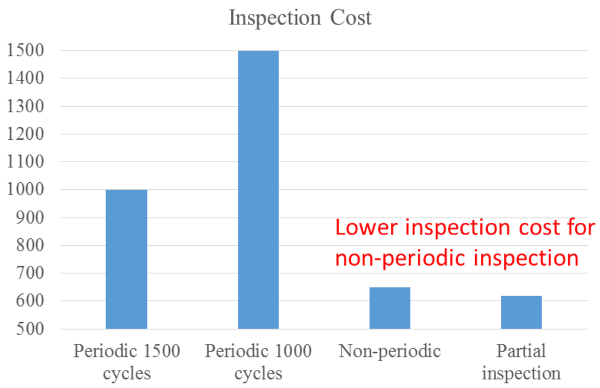


Fig. 4-2 Comparison of inspection cost of different inspection scheme

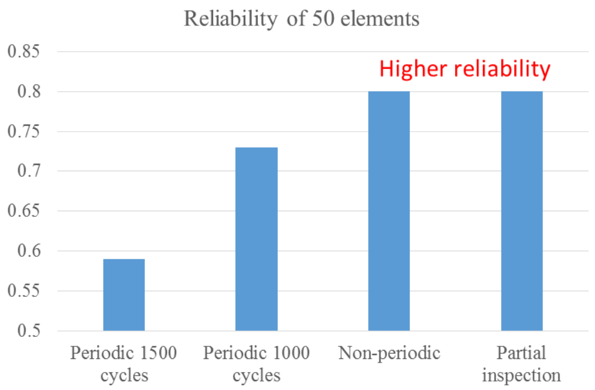


Fig. 4-3 Comparison of reliability of different inspection scheme

4.2. Effect of Bayesian method

Bayesian updating modified the distribution function of uncertain parameters according to the inspection results. That is to say, the result of one simulation is a casual result. It is necessary to performed statistical analysis to evaluate the effect of Bayesian approach using in this study.

4.2.1. Statistical analysis of different cases of uncertain parameters

In order to estimate the effective of Bayesian estimation of different cases of uncertain parameters, statistical analysis are performed. The average values and standard deviations of the inspection number and peak values of parameters are shown in Table 4-3 ~ Table 4-8. Different sample number from 500 to 2000 are used and results show that 1000 or more samples are enough to obtain good statistical analysis results for all cases.

For the case of uncertain parameter β, as shown in Table 4-3, the average result of inspection number from 2000 samples is slightly bigger than the number from true value. The average peak value of parameter β gives a good estimation of the true value. However, the standard deviation of inspection number is large which means the distribution range of the results of inspection number is very wide. The results are statically good but affect by accidental factor easily.

For the case of uncertain parameter c, as shown in Table 4-4, the average result of inspection number from 2000 samples is smaller than the number from true value. The average peak value of parameter c is 20% larger than the true value, thus an underestimate happens to the inspection number. The Bayesian estimation falls into a local minimum in the case of uncertain parameter c.

For the case of uncertain parameter d, as shown in Table 4-5, the average result of inspection number from 2000 samples is coincident with the number from true value. The estimation of parameter d is not good enough as the standard deviation is larger than 25%. However, because parameter d does not affect the reliability at all, the result of inspection number is extremely good.

For the case of two uncertain parameters, as shown in Table 4-6 , Table 4-7 and Table 4-8, results change depending on which parameter is uncertain. When parameter β is uncertain, the standard deviation of inspection number appears to be too large. When parameter c is uncertain, the inspection number will be underestimated. When parameter d is uncertain, the estimation of parameter d is not good enough. In the case of multi-parameters, two or three above problems will arise. Considering the local minimum problem and the lack of information, it should be concluded that the problem of multiple uncertain parameters is hard to solve.

Table 4-3 Statistical results for different number of sample (uncertain parameter β)

Number of sample	Number of inspections Average	SD	Parameter β Average	SD
500	13.62	2.72	40900	4580
	13.49	2.66	41260	4560
	13.66	2.94	41140	4980
1000	13.56	2.69	41080	4580
	13.65	2.92	41100	5000
2000	13.60	2.81	41080	4800
True value	12.98	0.17	40000	---

Table 4-4 Statistical results for different number of sample (uncertain parameter c)

Number of sample	Number of inspections Average	SD	Parameter c Average	SD
500	11.59	1.61	1.972E-04	2.96E-05
	11.71	1.69	1.964E-04	2.77E-05
	11.64	1.53	1.957E-04	2.70E-05
1000	11.65	1.65	1.968E-04	2.86E-05
	11.66	1.61	1.959E-04	2.84E-05
2000	11.66	1.63	1.964E-04	2.85E-05
True value	12.98	0.17	1.6E-04	---

Table 4-5 Statistical results for different number of sample (uncertain parameter d)

Number of sample	Number of inspections Average	SD	Parameter d Average	SD
500	12.79	0.49	39.02	11.28
	12.78	0.48	38.38	11.08
	12.78	0.51	38.54	10.78
1000	12.78	0.48	38.7	11.18
	12.79	0.51	39.14	11.16
2000	12.79	0.49	38.92	11.18
True value	12.98	0.17	40	---

Table 4-6 Statistical results for different number of sample (uncertain parameter β and c)

Number of sample	Number of inspections Average	SD	Parameter β Average	SD	Parameter c Average	SD
500	11.48	2.04	42580	5140	2.040E-04	2.65E-05
	11.51	1.98	42340	4940	2.041E-04	2.40E-05
	11.51	2.01	42380	4860	2.037E-04	2.52E-05
1000	11.49	2.01	42460	5040	2.041E-04	2.53E-05
	11.52	2.03	42500	5000	2.025E-04	2.59E-05
2000	11.51	2.02	42480	5020	2.033E-04	2.56E-05
True value	12.98	0.17	40000	---	1.6E-04	---

Table 4-7 Statistical results for different number of sample (uncertain parameter β and d)

Number of sample	Number of inspections Average	SD	Parameter β Average	SD	Parameter d Average	SD
500	13.10	2.56	41220	4960	39.5	11.8
	13.48	2.92	41160	4900	38.6	11.24
	13.33	2.75	41200	4940	38.58	11.14
1000	13.29	2.75	41200	4940	39.06	11.54
	13.35	2.69	41200	4880	38.1	11.22
2000	13.32	2.72	41200	4920	38.58	11.38
True value	12.98	0.17	40000	---	40	---

Table 4-8 Statistical results for different number of sample (uncertain parameter c and d)

Number of sample	Number of inspections Average	SD	Parameter c Average	SD	Parameter d Average	SD
500	11.15	1.44	2.107E-04	3.08E-05	31.52	10.9
	11.38	1.57	2.078E-04	3.28E-05	32.56	11.36
	11.27	1.51	2.098E-04	3.15E-05	31.7	11.5
1000	11.26	1.51	2.092E-04	3.18E-05	32.04	11.14
	11.21	1.51	2.100E-04	3.23E-05	31.88	11.32
2000	11.24	1.51	2.096E-04	3.21E-05	31.96	11.24
True value	12.98	0.17	1.6E-04	---	40	---

Table 4-9 Statistical results of error form Bayesian estimation (2000 samples)

Uncertain Parameters	No. of inspections Average	Error	Estimated β Average	Error	Estimated c Average	Error	Estimated d Average	Error
True value	12.98	---	40000	---	1.60E-4	---	40	---
β	13.60	4.8%	41080	2.7%	---	---	---	---
c	11.66	-10.2%	---	---	1.96E-04	22.8%	---	---
d	12.79	-1.5%	---	---	---	---	38.92	-2.7%
β and c	11.51	-11.3%	42480	6.2%	2.03E-04	27.1%	---	---
β and d	13.32	2.6%	41200	3.0%	---	---	38.58	-3.6%
c and d	11.24	-13.4%	---	---	2.10E-04	31.0%	31.96	-20.1%

Statistical results of error form Bayesian estimation for all cases are summarized in Table 4-9. Concerning the choice and effective of uncertain parameters, conclusions are made as follows:

1. It is better to have only one uncertain parameter than multiple uncertain parameters.
2. Parameter d should be treat as fixed parameter other than uncertain one because it does not affect the inspection interval and also easy to decided.
3. When parameter β is treated as uncertain, average of the estimations are good. Both number of inspection and parameter itself are estimated within 5% error.
4. When parameter c is treated as uncertain, local minimum problem exists. More information is necessary to improve the Bayesian estimation.

4.2.2. Statistical analysis of different prior density functions

The inspection intervals predicted by presented method depend on the distribution density functions of uncertain parameters. Especially for the first several inspections when there is no enough information to estimate the proper range of parameters, the prior density function dominates the length of the inspection intervals. There are two cases of the prior knowledge for parameters: one is that only lower bound and upper bound are known, the prior density function is a uniform distribution; the other is that the center value is known, and the prior density function is a normal distribution has a mean value known.

The statistical results of four types of prior density functions for uncertain parameter β are compared in Table 4-10. The upper three rows are the results when parameter β has a single value, and the lower four rows are the results when Bayesian method is used to update the distribution functions. Considering the true value of parameter is β = 40000, the right answer of number of inspection should be 12.98. When wrong value of parameter β is used, the errors are larger than 15% as shown in row 2 and 3. One the other hand, when Bayesian updating is applied, the wrong prior knowledge is revised and the errors are smaller than 5% except one case. The peak value of parameter β is estimated in good accuracy for all cases.

The statistical results of four types of prior density functions for uncertain parameter c are compared in Table 4-11. The upper three rows are the results when parameter c has a single value, and the lower four rows are the results when Bayesian method is used to update the distribution functions. Considering the true value of parameter is c = 1.6E-4, the right answer of number of inspection should be 12.98. When wrong value of parameter c is used, the errors are larger than 12% as shown in row 2 and 3. One the other hand, when Bayesian updating is applied, the wrong prior knowledge is revised and the errors are smaller than 12%. The peak value of parameter c is overestimated in all cases because of the local minimum problem. As a result of that, the number of inspection is underestimated. However, improvements are still can be found by using Bayesian method even when prior knowledge is wrong.

Table 4-10 Effect of prior information (Uncertain parameter β, 2000 samples)

Conditions	Number of inspections		Peak of estimated β	
	Average	Error	Average	Error
True value β = 40000	12.98	---	---	---
Wrong value β = 36000	16.35	26.0%	---	---
Wrong value β = 44000	10.66	-17.9%	---	---
Normal dist. μ = 36000	14.59	12.4%	40060	0.2%
Normal dist. μ = 40000	13.56	4.5%	40460	1.2%
Normal dist. μ = 44000	12.52	-3.5%	41180	3.0%
Uniform distribution of β	13.60	4.8%	41080	2.7%

Table 4-11 Effect of prior information (Uncertain parameter c, 2000 samples)

Conditions	Number of inspections		Peak of estimated β	
	Average	Error	Average	Error
True value c = 1.60E-4	12.98	---	---	---
Wrong value c = 1.4E-4	14.96	14.9%	---	---
Wrong value c = 1.8E-4	11.4	-12.2%	---	---
Normal dist. μ = 1.4E-4	12.57	-3.2%	1.77E-04	10.6%
Normal dist. μ = 1.6E-4	12.04	-7.2%	1.83E-04	14.2%
Normal dist. μ = 1.8E-4	11.43	-11.9%	1.90E-04	18.4%
Uniform distribution of c	11.66	-10.2%	1.96E-04	22.8%

4.2.3. Effect of Bayesian method when parameters unknown

Bayesian method can solve the problem when parameters are uncertain. The effect of Bayesian method when parameters are unknown is evaluated by comparing the estimated number of inspections in six cases to the "right answer" when all parameters are fixed.

The comparison of inspection number (unknown parameter β) is shown in Fig. 4-4. A ±10% range beside the right answer when β equal to right value 40000 is indicated in the figure. The answers when a wrong value of β is used are out of ±10% range, while Bayesian method can obtain an answer within 10% error. The comparison of inspection number (unknown parameter c) is shown in Fig. 4-5. A ±10% range beside the right answer when c equal to right value 1.6E-4 is indicated in the figure. The answers when a wrong value of c is used are out of ±10% range. The Bayesian method gives answers which are slightly underestimated but still acceptable when parameter c is unknown.

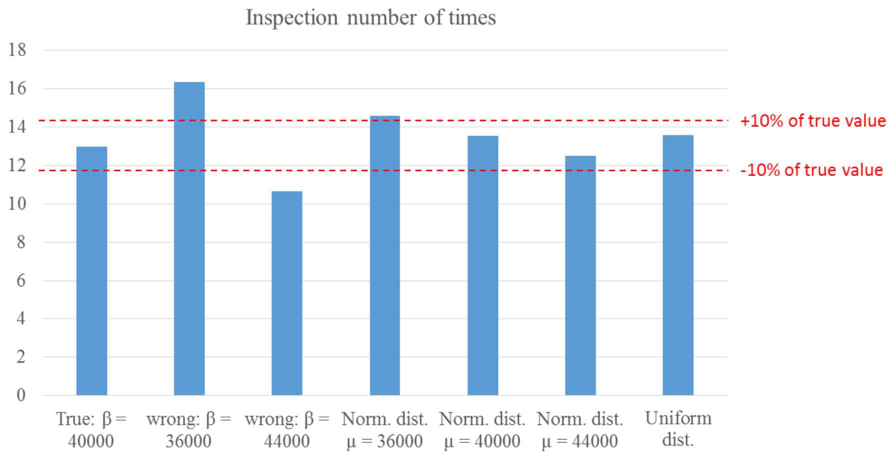


Fig. 4-4 Comparison of inspection number (True, wrong values and Bayesian method for β)

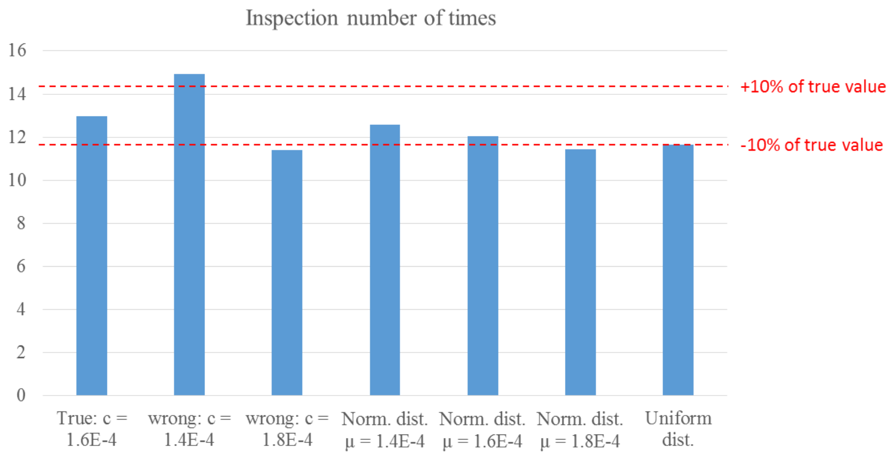


Fig. 4-5 Comparison of inspection number (True, wrong values and Bayesian method for c)

4.3. Reliability of estimated reliability

The reliabilities discussed in this study is an estimated probability results. It is necessary to evaluate the estimated reliability by a means of reliability evaluation.

4.3.1. Evaluation of estimated reliability

The estimated reliability is the reliability computed by knowing only inspection results using a probability method. Because the "truth" is a simulated virtual result, one can computed the true reliability of the system by knowing initiation information of fatigue cracks. Fig. 4-6 shows the comparison of estimated reliability and true reliability of entire system by simulation results with all parameters fixed. While all estimated reliability kept above 0.8 throughout the service life, the true reliability varied sometimes as low as 0.7.

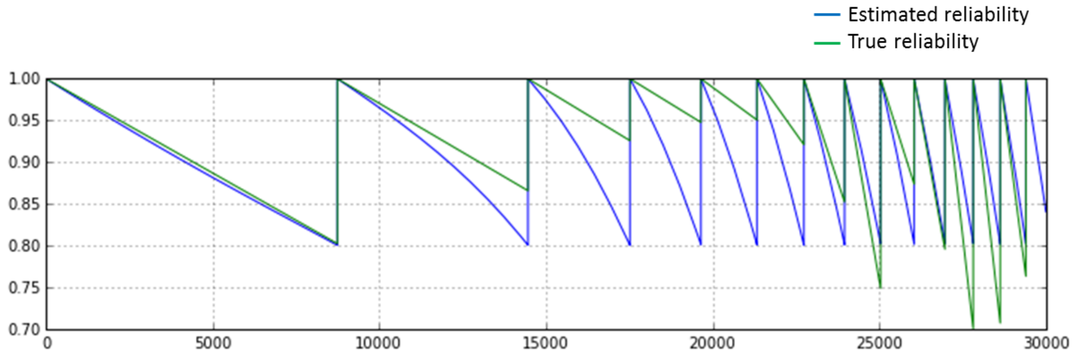


Fig. 4-6 Comparison of the estimated reliability and true reliability (True parameters)

4.3.2. Statistical analysis for reliability of estimated reliability

A parameter is defined as the reliability of estimated reliability (Ror.) at each inspection points as follows:

$$\mathrm{Ror.} = \begin{cases} 1 & \text{when } R_{\mathrm{true}} \geq R_{\mathrm{design}} 0.8 \\ R_{\mathrm{true}}/R_{\mathrm{design}} & \text{when } R_{\mathrm{true}} < R_{\mathrm{design}} 0.8 \end{cases} . \tag{4-1}$$

Several cases of simulations are performed including: (1) True value of parameters, (2) True value of parameters by partial inspection scheme, (3) Bayesian estimation of parameter β, (4) Bayesian estimation of parameter β by partial inspection scheme, (5) Bayesian estimation of parameter β and c, and (6) Bayesian estimation of parameter β and c by partial inspection scheme. The reliabilities of estimated reliability (Ror.) are shown in Table 4-12 and Fig. 4-7. It is clear that no matter using true value of parameters or using Bayesian estimation for uncertain parameters, the average of Ror. keeps above 0.9 and the minimum of Ror. keeps above 0.6. Also, partial inspection scheme does not influence the reliability.

Table 4-12 Statistical results of reliability of estimated reliability (2000 samples)

Conditions	Average of Ror.		Minimum of Ror.	
	Mean value	SD	Mean value	SD
True values	0.925	0.052	0.656	0.185
True values, partial Insp.	0.924	0.052	0.654	0.184
Bayesian β	0.928	0.038	0.617	0.161
Bayesian β, partial Insp.	0.931	0.036	0.624	0.159
Bayesian β,c	0.904	0.057	0.610	0.172
Bayesian β,c, partial Insp.	0.905	0.054	0.607	0.169

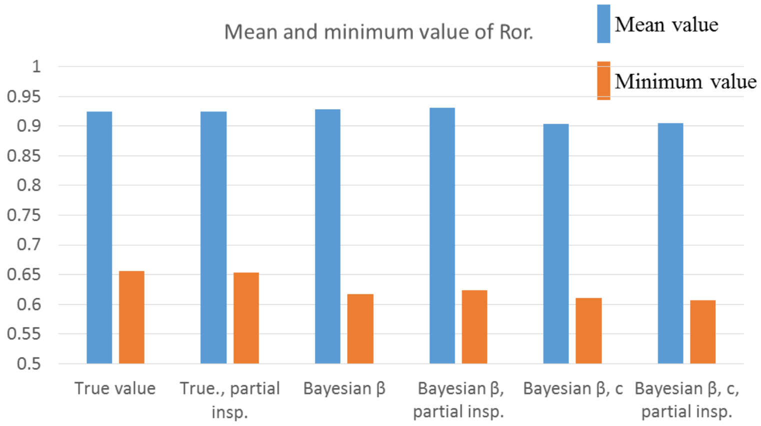


Fig. 4-7 Comparison of reliability of estimated reliability (Ror.)

4.4. Summary

The advanced Bayesian approach is introduced to optimize the non-periodic inspection intervals. Statistical analysis are performed to evaluate the effect of the approach and the Bayesian updating. Contents and results are summarized as follows:

1. It is possible to greatly reduce the system expected total cost by optimize the non-periodic inspection intervals using Bayesian approach, comparing with periodic inspection.
2. The selection of uncertain parameters is important and single uncertain parameter is preferred. Bayesian method works better if more information is available.
3. Comparison between estimated reliability and true reliability is performed by statistical analysis. The reliability of the estimated reliability is larger than 0.9. The presented approach is applicable to optimization of inspection intervals because of its high reliability.

5. Bayesian method using conditional probability

In chapter 3, the prior density function is supposed to be the possible distribution at the time of the initiation of service and the likelihood function of an element is considered from service start. This is not convenient when not all the historical information of inspections are available and simulation from the middle of a service life is needed. Bayesian analysis method using conditional probability can solve this problem. It is not necessary to acquire all information of inspections from the service starts when using conditional probability. However, the information from the time of last repair or replacement of the element are still necessary.

5.1. Conditional probability and reliability

The conditional probability as well as the reliability computations are mostly the same as chapter 2 with minor change in forms.

5.1.1. The probabilities in the condition of repaired at (j-1)-th inspection

(1) Event A^Y and probabilities of that a failure was found at j-th inspection

Probability of event A^Y can be derived from eq. (2-23) as follows:

$$\begin{aligned} P\{A^Y\!: j|\mathrm{Rep}^{j-1}\} = & \{1 - F_c(T_j - T_{j-1}|\beta)\}\cdot\{1 - U(T_j - T_{j-1})\} \\ & + \int_{T_{j-1}}^{T_j} f_c(t - T_{j-1}|\beta)\cdot\{1 - U(t - T_{j-1})\}\mathrm{d}t \\ & + \int_{T_{j-1}}^{T_j} f_c(t - T_{j-1}|\beta)\cdot U(t - T_{j-1})\cdot\{1 - V(T_j - t)\}\mathrm{d}t\,, \end{aligned} \tag{5-1}$$

where Rep^{j-1} represents the event that the element was repaired at (j-1)-th inspection.

(2) Event B^Y and probabilities of that a crack was found at j-th inspection

Probability of event B^Y can be derived from eq. (2-24) as follows:

$$P\{B^Y(a_j)\!: j|\mathrm{Rep}^{j-1}\} = f_c(t_c - T_{j-1}|\beta)\left|\frac{\mathrm{d}t_c}{\mathrm{d}a_j}\right|\Delta a\cdot U(t_c - T_{j-1})\cdot V(T_j - t_c)\cdot D(a_j|d). \tag{5-2}$$

(3) Event C^Y and probabilities of that nothing was found at j-th inspection

Probability of event C^Y can be derived from eq. (2-25) as follows:

$$P\{C^Y : j|\mathrm{Rep}^{j-1}\} = \{1 - F_c(T_j - T_{j-1}|\beta)\} \cdot U(T_j - T_{j-1}) + \int_{T_{j-1}}^{T_j} f_c(t - T_{j-1}|\beta) \cdot U(t - T_{j-1}) \cdot V(T_j - t) \cdot \{1 - D(a(T_j - t|c)|d)\}\mathrm{d}t\,. \tag{5-3}$$

5.1.2. The probabilities in the condition of not repaired at (*j*-1)-th inspection

(1) Event A^N and probabilities of that a failure was found at *j*-th inspection

Probability of event A^{Y} satisfied the following equation:

$$P\{A^N : j, l|\mathrm{No}^{j-1}, l\} = \frac{P\{A^N : j, l \cap \mathrm{No}^{j-1}, l\}}{P\{\mathrm{No}^{j-1}, l\}}\,, \tag{5-4}$$

where No^{j-1} represents the event that the element was not repaired at (*j*-1)-th inspection, and *l* shows that last repair or replacement occurred at T_l. The numerator $P\{A^N : j, l \cap \mathrm{No}^{j-1}, l\}$ of eq. (5-4) has the same form as eq. (2-23) $P\{A : j, l\}$ but with the restriction that $l \neq (j-1)$. The denominator $P\{\mathrm{No}^{j-1}, l\}$ of eq. (5-4) can be computed as $P\{C : j-1, l\}$ from eq. (2-25).

(2) Event B^N and probabilities of that a crack was found at *j*-th inspection

Probability of event B^{Y} satisfied the following equation:

$$P\{B^N(a_j) : j, l|\mathrm{No}^{j-1}, l\} = \frac{P\{B^N(a_j) : j, l \cap \mathrm{No}^{j-1}, l\}}{P\{\mathrm{No}^{j-1}, l\}}\,. \tag{5-5}$$

The numerator $P\{B^N(a_j) : j, l \cap \mathrm{No}^{j-1}, l\}$ of eq. (5-5) has the same form as eq. (2-24) $P\{B(a_j) : j, l\}$ but with the restriction that $l \neq (j-1)$.

(3) Event C^N and probabilities of that nothing was found at *j*-th inspection

Probability of event C^{Y} satisfied the following equation:

$$P\{C^N : j, l|\mathrm{No}^{j-1}, l\} = \frac{P\{C^N : j, l \cap \mathrm{No}^{j-1}, l\}}{P\{\mathrm{No}^{j-1}, l\}}\,. \tag{5-6}$$

The numerator $P\{C^N : j, l \cap \mathrm{No}^{j-1}, l\}$ of eq. (5-6) has the same form as eq. (2-25) $P\{C : j, l\}$ but with the restriction that $l \neq (j-1)$.

5.1.3. Reliability computation

The reliability of elements at time instant t^* after time of inspection T_j, can be calculated the same as shown in eq. (2-26) and (2-27).

5.2. Likelihood function and posterior probability density function

5.2.1. Likelihood function

The likelihood function for Bayesian analysis method using conditional probability needs only the information of inspection from last repair or replacement. Thus eq. (3-5), the likelihood function for element *m* resulting from the *j*-th inspection is changed to:

$$LF_j^{(m)}E^{j-1} = P_m\{Y{:}\, j|\mathrm{Rep}^{j-1}\} \text{ or } P_m\{X{:}\, j, l|\mathrm{No}^{j-1}, l\}. \tag{5-7}$$

In eq. (5-7), *Y* stands for either even A^Y, B^Y or C^Y, *X* stands for either A^N, B^N or C^N depending on the result of the *j*-th inspection for element *m*. E^{j-1} stands for either "Replace" or "No replace" at time of (*j*-1)-th inspection. The probability $P_m\{*\}$ refers to eq. (5-1) to (5-6).

The likelihood function LF_j for the entire system as a result of the *j*-th inspection is calculated as:

$$LF_jE^{j-1} = \prod_{m=1}^{M} LF_j^{(m)}E^{j-1}\,, \tag{5-8}$$

where *M* is the total number of elements in the system.

5.2.2. Posterior probability density function

The posterior joint density function of parameters *β*, *c* and *d* after the *j*-th inspection is given by:

$$f^j(\beta, c, d) = \frac{LF_jE^{j-1} \cdot f^{j-1}(\beta, c, d)}{\int_{\beta_{min}}^{\beta_{max}} \int_{c_{min}}^{c_{max}} \int_{d_{min}}^{d_{max}} (\mathrm{Numerator})\mathrm{d}\beta \mathrm{d}c \mathrm{d}d}\,. \tag{5-9}$$

It is clear that likelihood function LF_j is conditional to given values of *β*, *c* and *d*.

5.3. Numerical results

Numerical simulation is performed using the conditional probability approach. The inspection schedule are shown in Table 5-1. Reliabilities for the non-periodic inspections are shown in Fig. 5-1, where figure (a) is the reliability of a single element and (b) is the reliability of entire system with 50 elements. The result is basically the same as the non-periodic inspection scheme in chapter 2 because they are the answer to a same problem.

By using conditional probability, the Bayesian approach can be applied not only from the service initiation but also from the middle of service. It gives more flexibility to this method such as that the continuity of the Bayesian updating can be applied more easily from different systems. That is to say, when two systems are working in exactly same condition, Bayesian updating may be applied interactively between two systems. More information from inspections will improve the Bayesian updating and its convergence.

Examples are shown by applying Bayesian updating in four sequent systems. For simplification, four systems are applied in order instead of interactively. The last posterior density function of the first systems is used as the prior density function of the second system. And then the posterior density function of second system is transferred to the third system, then to the forth.

The change of probability distribution function of uncertain parameter β is shown in Fig. 5-2. The results of inspection number and peak value of parameter after Bayesian estimation from each system is listed in Table 5-2. The probability distribution function converges to the true value better and more concentrate after multi-system updating. Due to the concentration of distribution function of parameter to the true value, the inspection number converge to statistic average results of true value.

The change of probability distribution function of uncertain parameter c is shown in Fig. 5-3. The results of inspection number and peak value is listed in Table 5-3. The estimated results of parameter c falls into a distribution with local minimum and leads to a wrong inspection number.

The change of probability distribution function of uncertain parameter d is shown in Fig. 5-4. The results of inspection number and peak value is listed in Table 5-4. Different from the case of parameter c, the probability distribution function of parameter d converges and concentrates to the true value slower but no local minimum problem occurs. The estimated inspection number gives a good result.

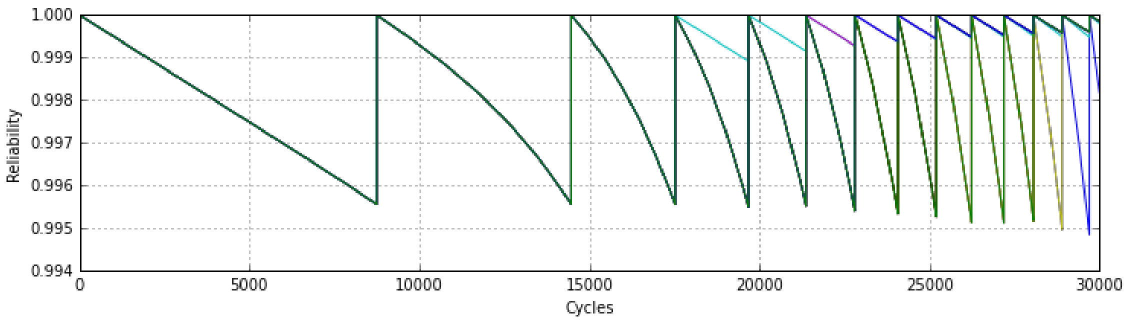
5.4. Summary

The presented approach in chapter 2 and 3 is improved by introducing conditional probability. Both approaches give identical result but the improved method shows more flexibility.

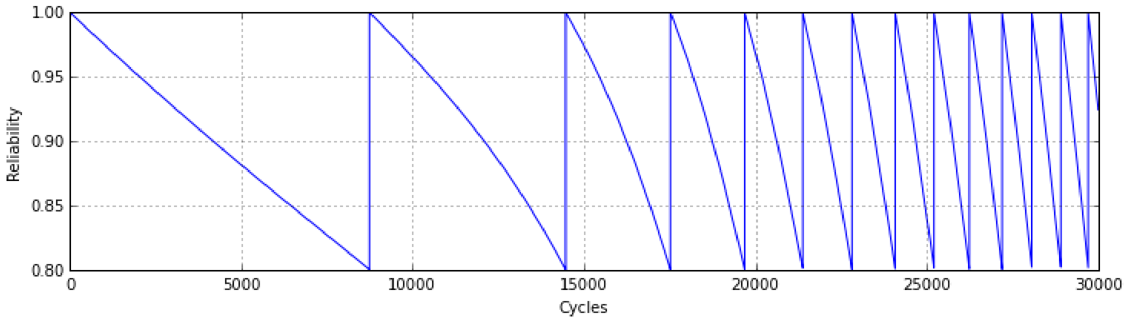
1. The method is applied to series multi-system to check the effect of increasing number of Bayesian updating.
2. Results show that parameter β and d converge better when number of Bayesian updating increases.
3. On the other hand, parameter c falls into local minimum and the number of Bayesian updating does not solve the problem.

Table 5-1 Inspection schedule of non-periodic scheme by conditional probability (true value)

Inspection no.	Inspection time (cycles)	Inspection interval (cycles)
1	8740	8740
2	14460	5720
3	17520	3060
4	19680	2160
5	21370	1690
6	22810	1440
7	24070	1260
8	25200	1130
9	26240	1040
10	27190	950
11	28060	870
12	28900	840
13	29700	800



(a) Reliability of a single element

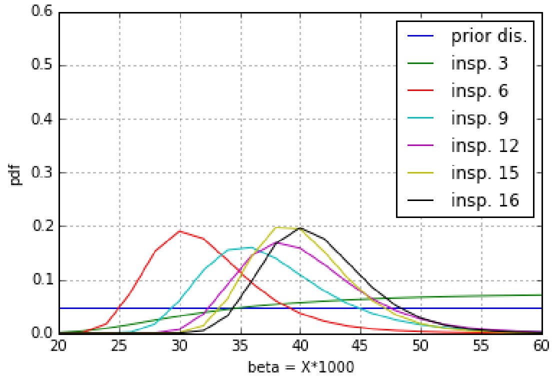


(b) Reliability of entire system with 50 elements

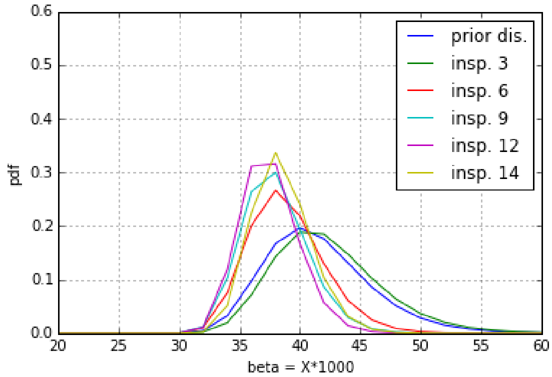
Fig. 5-1 Reliability for the non-periodic inspection by conditional probability

Table 5-2 Bayesian estimation after multi-system life (uncertain parameter β)

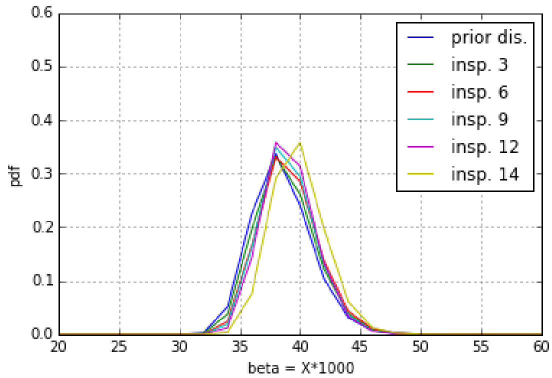
System no.	Number of Inspections	Peak value of parameter β
1	16	40000
2	14	38000
3	14	40000
4	13	40000
True value	12.98	40000



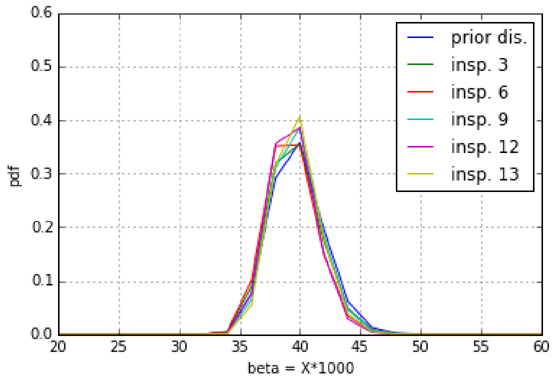
(1) System no. 1



(2) System no. 2



(3) System no. 3

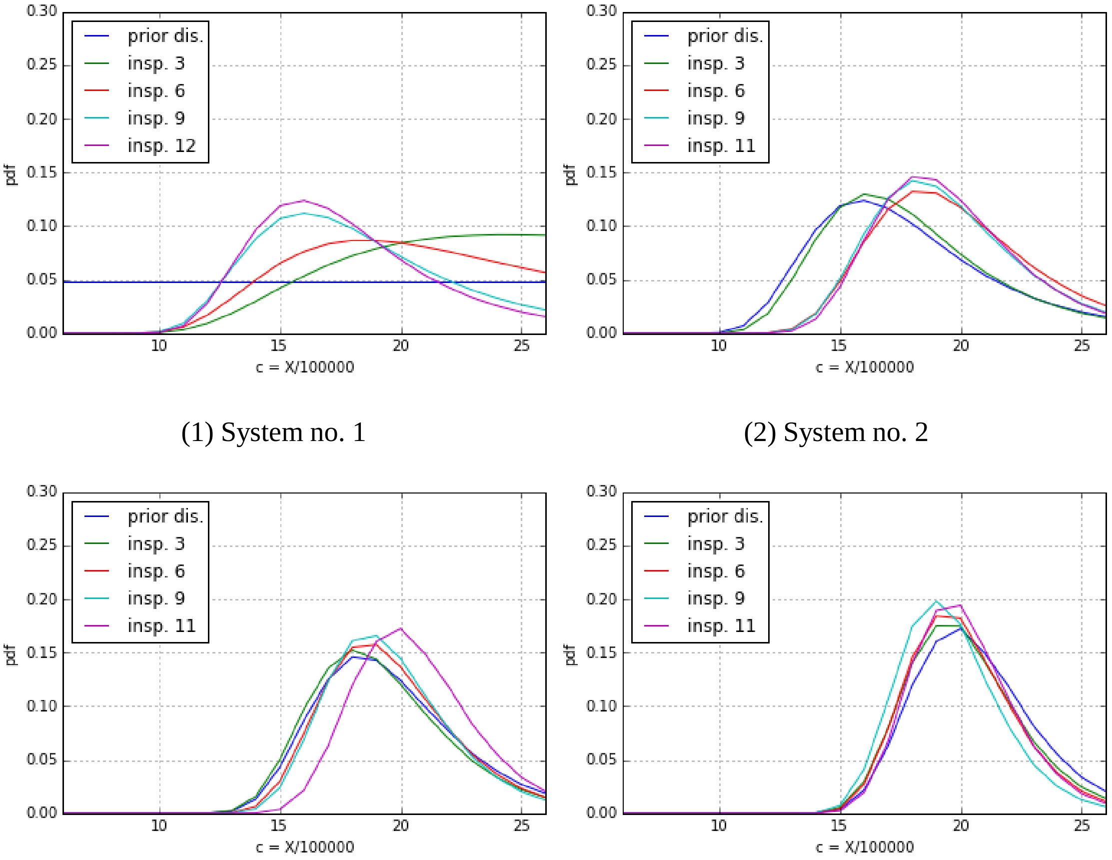


(4) System no. 4

Fig. 5-2 Change of probability distribution function (uncertain parameter β)

Table 5-3 Bayesian estimation after multi-system life (uncertain parameter c)

System no.	Number of Inspections	Peak value of parameter c
1	12	1.600E-4
2	11	1.800E-4
3	11	2.000E-4
4	11	2.000E-4
True value	12.98	1.600E-4



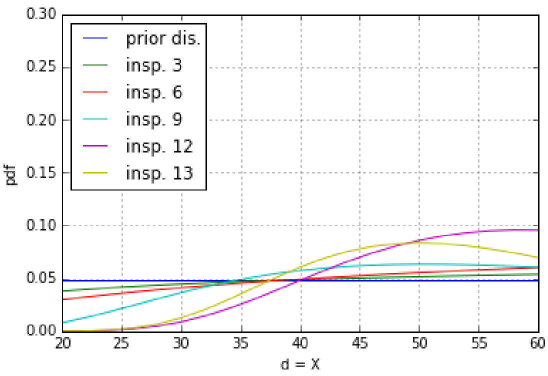
(1) System no. 1 (2) System no. 2

(3) System no. 3 (4) System no. 4

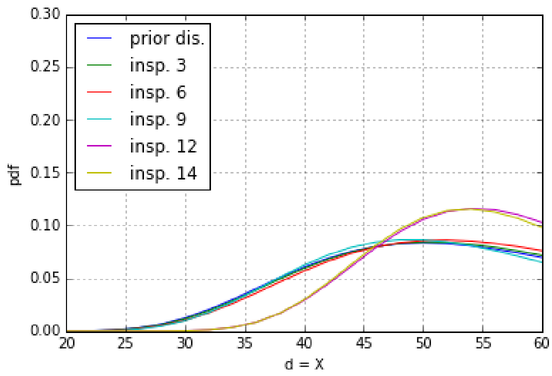
Fig. 5-3 Change of probability distribution function (uncertain parameter c)

Table 5-4 Bayesian estimation after multi-system life (uncertain parameter *d*)

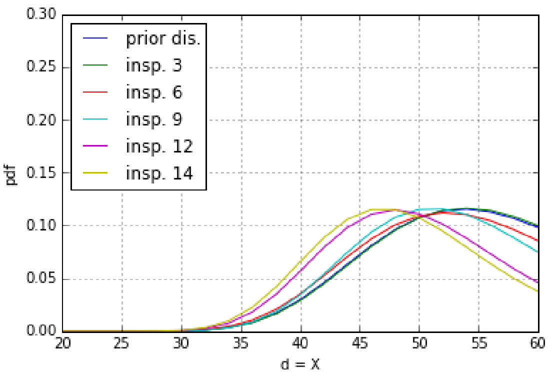
System no.	Number of Inspections	Peak value of parameter *d*
1	13	50
2	14	54
3	14	48
4	13	42
True value	12.98	40



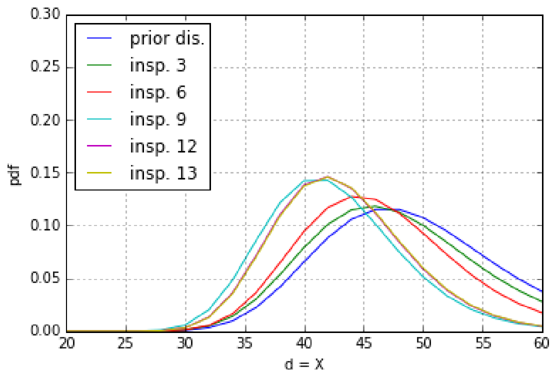
(1) System no. 1



(2) System no. 2



(3) System no. 3



(4) System no. 4

Fig. 5-4 Change of probability distribution function (uncertain parameter *d*)

6. Example of application

An example of application to the gas turbine engine components is shown in this chapter.

6.1. Retirement for cause

Retirement for cause (RFC) is a life cycle management procedure for gas turbine engine components, such as fan, compressor and turbine disks. The procedure enables full use of the safe life inherent in each component, as opposed to arbitrary retirement from service of all components at a calculated low cycle fatigue life. Historically, these components are retired when they reach an analytically determined lifetime where the first fatigue crack per 1000 disks could be expected. By definition then, 99.9 percent of these components were being retired prematurely, while they still may have had useful life remaining.

The retirement for cause approach [3] is based on fracture mechanics and nondestructive evaluation (NDE). When components reach the analytically determined lifetime, nondestructive inspection is applied. Only components with cracks are retired, as opposed to arbitrary retirement from service of all components. The analytically determined lifetime, the time when a detectable crack is found in 1/1000 probability, is the interval to the first inspection. All intact components are return to service after inspection. Fig. 6-1 is a figure pickup from reference [3], which shows the base retirement for cause concept. The safe return-to-service intervals is determined by the propagation time between NDE limit length and critical length of failure.

As described above, RFC procedure is a deterministic method depended on the prior knowledge of initiation and propagation of fatigue cracks. Difficulties are encountered in cases where the parameters are uncertain.

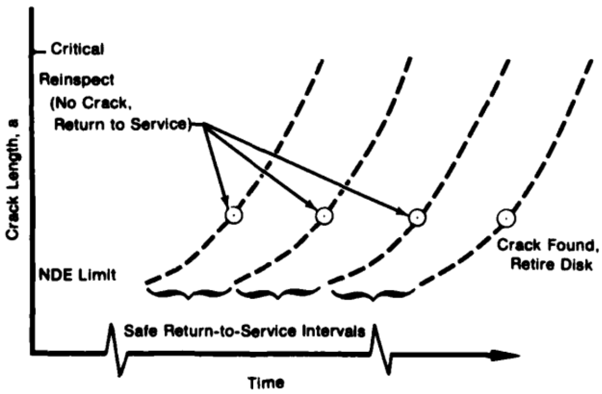


Fig. 6-1 Base retirement for cause concept (from ref. [3])

6.2. Estimation of parameters

The Bayesian approach is applied to the gas turbine engine disks. The objective of the application is to improve the RFC procedure by non-periodic inspection and Bayesian method. RFC procedure is a deterministic method with all parameters are given. On the other hand, Bayesian method introduces some uncertain parameters and updates their joint density function after inspection results are available.

According to the discussions in the former chapters, Bayesian updating of multiple uncertain parameters works not so good because of local minimum and lack of information. A simplified Bayesian method with single uncertain parameter is applied to the problem of turbine engine. In order to obtain an inspection scheme similar to RFC procedure, some minor modifications are done to the normal Bayesian method as shown in Table 6-1. It should be pointed out that RTS interval is the most important parameter for RFC. That is the reason why parameter c in the propagation function of the fatigue cracks is selected to be uncertain in our discussion.

Table 6-1 Comparison of RFC, normal Bayesian method and application to turbine engine

	RFC (Retirement for cause)	Normal Bayesian method	Bayesian method applied to turbine engine
Initiation of cracks	Deterministic, affect only the first inspection	Consider as probability, parameter β may be uncertain	Consider as probability, parameter β fix to true value
Propagation of cracks	Deterministic, affect the RTS intervals	Consider as probability, parameter c may be uncertain	Consider as probability, parameter c may be uncertain
Detectability	May consider, normally assume to be 100%	Consider as probability, parameter d may be uncertain	Consider as probability, parameter d fix to true value
Failure rate	Depend on crack length, critical length set	Before initiation: random After initiation: function of time (cycles)	Before initiation: random After initiation: function of crack length

6.2.1. Initiation of a fatigue crack

All fatigue data have inherent scatter. The data base used for design life analyses purposes must be applicable to all disks of a given material, and therefore includes test results from many heats and sources. According to reference [3], data are treated statistically as shown in Fig. 6-2.

The time to crack initiation (TTCI) is assumed to be a random variable with density function following the Weibull distribution as shown in eq. (2-1). Copy as follows:

$$f_c(t|\beta) = \frac{\alpha}{\beta} \cdot \left(\frac{t}{\beta}\right)^{\alpha-1} \cdot exp\left[-\left(\frac{t}{\beta}\right)^{\alpha}\right] \qquad t > 0. \tag{6-1}$$

The parameters for gas turbine engine disks are chosen as $\alpha = 2$ and $\beta = 60000$. The probability density function of the initiation of a fatigue crack is shown in Fig. 6-3. Under the conditions of $\alpha = 2$ and $\beta = 60000$, the curves in Fig. 6-2 and Fig. 6-3 are similar.

It is noted that the number of cycles to initiate a fatigue crack in a 1/1000 probability is 1897. The initiation crack length is assumed to be 0.25mm in Fig. 6-3, while the crack length in Fig. 6-2 is supposed to be 0.03inch = 0.762mm.

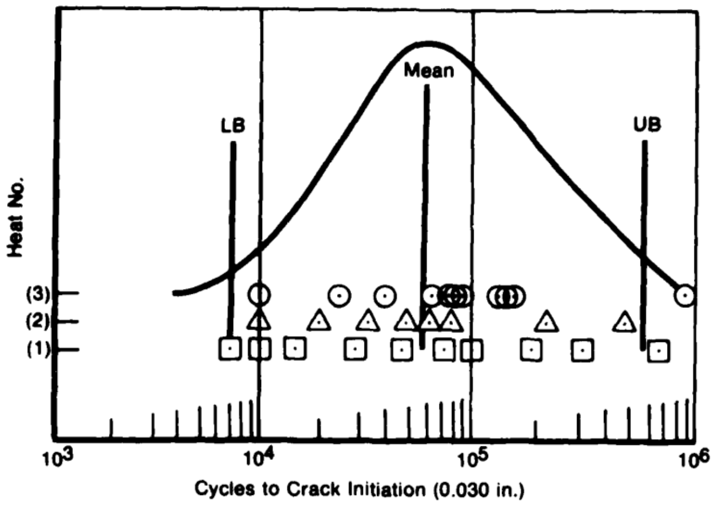


Fig. 6-2 Material data scatter results in conservative life prediction (from ref. [3])

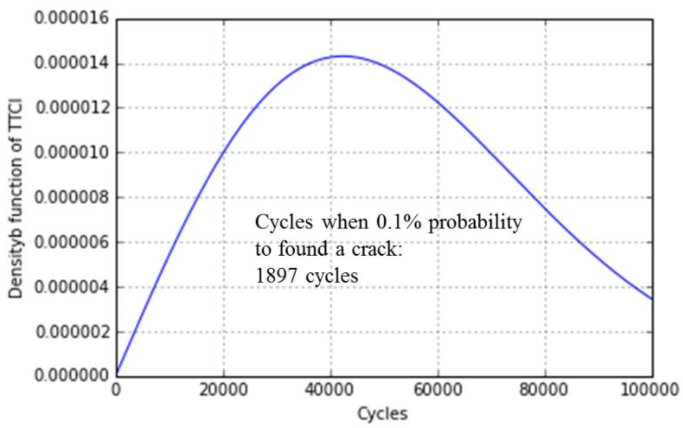


Fig. 6-3 Probability density function of the initiation of a fatigue crack (0.25mm)

6.2.2. Propagation of a fatigue crack

Fracture mechanics theory is used to determine the length of a propagating crack under random stress. It is assumed that a crack grows according to eq. (2-3) and (2-4). Copy eq. (2-4) as follows:

$$a(t - t_c|c) = \left[-b'c(t - t_c) + a_0{}^{-b'}\right]^{-1/b'} \quad \text{where } b' = \frac{b-2}{2}\ . \tag{6-2}$$

The parameters for gas turbine engine disks are chosen as $b = 2.96$ and $c = 1.0 \times 10^{-3}$. The propagation curve of a fatigue crack is shown in Fig. 6-4. Refer to reference [3], the initial crack length is assumed to be $a_0 =$ 0.25mm, the detectable crack length is 0.5mm and the critical length is 1.5mm.

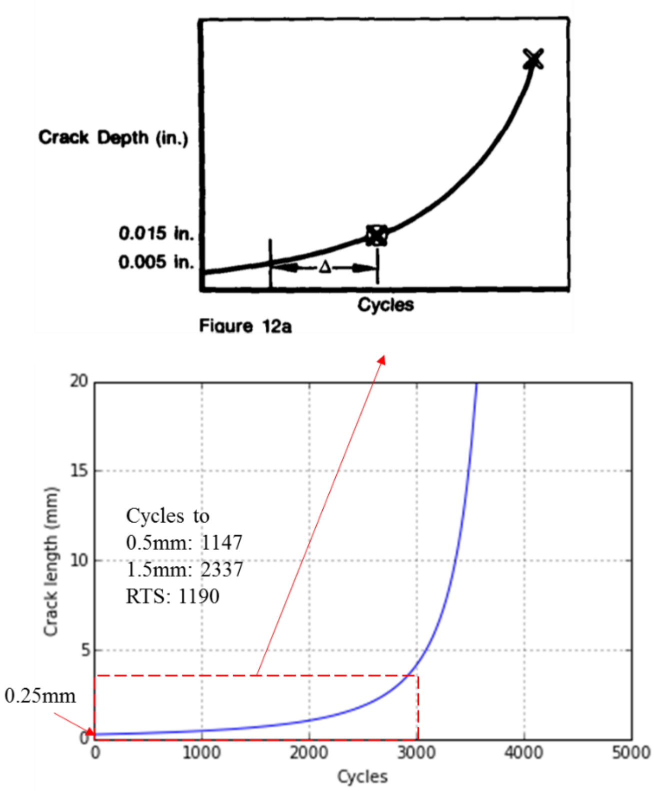


Fig. 6-4 Propagation of a fatigue crack (upper graph from ref. [3])

6.2.3. Probability of crack detection

The probability of detecting an existing crack (POD) of length *a* during an inspection is given by eq. (2-5). Copy as follows:

$$D(a|d) = 1 - exp\left[-\left(\frac{a - a_{min}}{d - a_{min}}\right)^{\theta}\right]. \tag{6-3}$$

The parameters for gas turbine engine disks are chosen as $\theta = 2$ and $d = 0.5$. The cracks shorter than $a_{min} =$ 0.25mm are supposed to be undetectable. The curve of probability of detection under these conditions is shown in Fig. 6-5. It is calculated that the detectability of 0.5 mm crack is 0.6321 and the detectability of 1.5 mm crack is 0.9999.

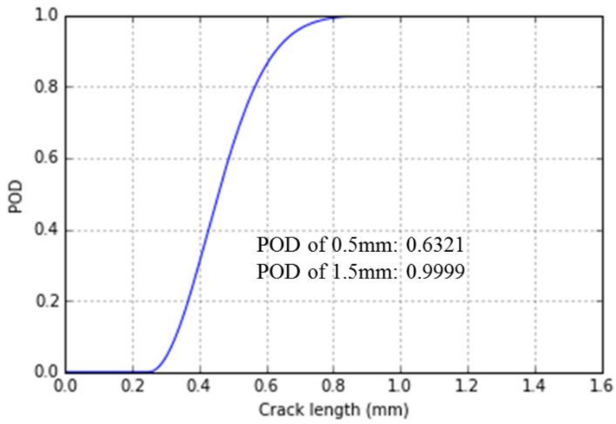


Fig. 6-5 Probability of detection

6.2.4. Reliability depending on crack length

Reliability after crack initiation is assumed to be a function of the crack length *a* as follows:

$$V_1(a) = exp\left\{-\left(\frac{a}{\beta_f}\right)^{\alpha_f}\right\}. \quad (6\text{-}4)$$

Because the crack length *a* is a function of time as eq. (6-2) shows, the equation of reliability can be rewrite as:

$$V_1(t - t_c) = exp\left\{-\left(\frac{a(t - t_c)}{\beta_f}\right)^{\alpha_f}\right\} \quad \text{for } t > t_c. \quad (6\text{-}5)$$

The critical length for a fatigue crack in engine disks is set to be 1.5mm as discussed above. The reliability when a 1.5mm crack exists is defined to be 0.5, and this condition (a 1.5mm length crack exists or conditional reliability 0.5 after crack initiation) is set as the minimum reliability level when planning the inspection scheme.

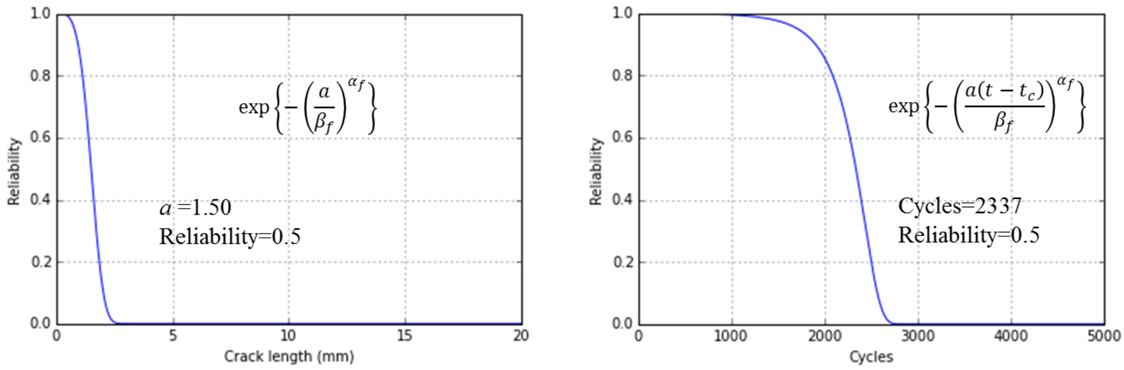
According to the condition that reliability is 0.5 when a 1.5mm crack exists, the parameters for gas turbine engine disks in eq. (6-4) and (6-5) are chosen as $\alpha_f = 3.7$ and $\beta_f = 1.656$. The reliability as a function of crack length or cycles is shown in Fig. 6-6. The number of cycles when reliability decreases to 0.5 is 2337. One thing must be remembered is that this reliability is under the condition of crack initiation. Considering the probability of crack initiation should be 1/1000 as required by RFC procedure, the true reliability is 0.9995 instead.

Different from RFC, the Bayesian method presented here can also consider the random failure before crack initiation during the service life. The whole service life is assumed to be 12000 cycles based on the data in reference [3]. Reliability from time instant T_l up to time instant t is denoted as U and given by eq. (2-9). Copy as follows:

$$U(t - T_l) = exp\{-(t - T_l) \cdot exp(r)\} \quad \text{for } t \leq t_c \,, \tag{6-6}$$

where T_l is the time of service initiation for the element, t_c is the time of crack initiation.

Assuming that the reliability U in whole service life will decrease to nearly 0.9995 (the same as a 1.5mm crack occurs), the parameter r is chosen as -17.0. The cycles when reliability U becomes 0.9995 is 12800 as shown in Fig. 6-7.



(a) Reliability vs. crack length (b) Reliability vs. cycles

Fig. 6-6 Reliability after initiation of a fatigue crack

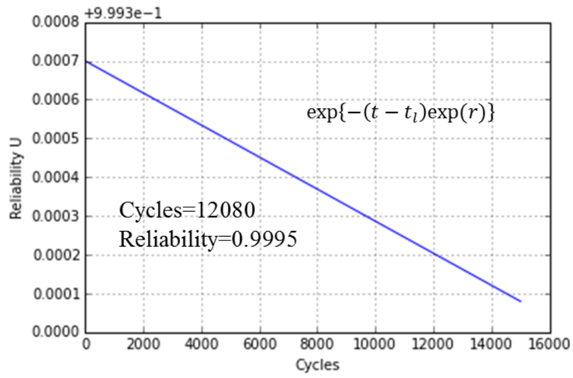


Fig. 6-7 Reliability before crack initiation (random failure)

Including the random failure shown in eq. (6-6), the reliability of an element after crack initiation during the service period from time instant of crack initiation t_c up to time instant t is denoted as V and given by:

$$V(t-t_c) = exp\left\{-\left(\frac{a(t-t_c)}{\beta_f}\right)^{\alpha_f} - (t-t_c)\cdot exp(r)\right\} \quad \text{for } t > t_c\,. \tag{6-7}$$

All parameters for gas turbine engine disks are summarized in Table 6-2. The design life limitation of disks is 12000 cycles which is also the service life of a turbine engine. To secure the safety of the safety-critical system turbine engine, not only the total reliability level but also the reliability level of a single element are considered. Total number of components (disks) in the system (engine) is selected to be 500 in this study.

Only the parameter c in the equation of crack propagation is considered as uncertain. The range is also shown in Table 6-2.

Table 6-2 Values of parameters in numerical simulation for engine disks

Item		True values	Uncertain range
General	Design life limitation	12,000 cycles	
	Minimum level of R_{single}	0.9995	
	Number of element M	500	
	Minimum level of R_{design}	$0.9995^{500} = 0.7788$	
Initiation:	Parameter α	2.0	
Eq. (6-1)	Parameter β	60,000 cycles	
Propagation:	Parameter b	2.96	
Eq. (6-2)	Parameter c	$1.0*10^{-3}mm^{-0.48}/cycle$	$0.5*10^{-3} \sim 1.5*10^{-3}$
	Initial crack length a_0	0.25mm	
Detectability:	Parameter a_{min}	0.25mm	
Eq. (6-3)	Parameter θ	2.0	
	Parameter d	0.5mm	
Reliability:	Parameter r	-17.0	
Eq. (6-6)	Parameter α_f	3.7	
Eq. (6-7)	Parameter β_f	1.656mm	

6.3. Non-periodic inspection intervals using true value of parameter

RFC procedure is a periodic inspection scheme except for the first time of inspection. On the other hand, the approach presented in this study is a non-periodic inspection method. The inspection scheme when all parameters are fixed to their true values are discussed in this section first.

6.3.1. The effect of number of components

Inspections scheme for turbine engine disks are optimized by presented approach. Because of the fact that not only the total reliability level but also the reliability level of single element are considered, number of the components does not affect the inspection scheme itself. Simulations are performed for one element and 500 element system.

The reliability of single element for both systems are shown in Fig. 6-8 and Fig. 6-9. The ● mark shown in the figures represents the replacement of one or more components. Both cases need 8 inspections for the whole service life, and the inspection schemes are exactly the same.

For the system with only one element, the reliability is mainly depend on the reliability function when no initiation of cracks (very small chance a fatigue crack occurs for only one element). For the system with 500 elements, some components are replaced due to cracks found but there are always some other elements remain un-replaced. The inspection scheme is decided by the un-replaced elements which means that it should be the same as one element system.

The intervals between each sequent inspections are shown in figures. Intervals become shorter as a function of the time. This is a logical consequence which is more reasonable than periodic inspection.

As a comparison, the normal RFC procedure is estimated as follows:

$$T_{first\ inspection} = T_{initiation} + \left(T_{propagation\ to\ 0.5mm} + T_{propagation\ to\ 1.5mm}\right)/2 = 1897 + (1147 + 2337)/2 = 3639 \text{ cycles,} \tag{6-8}$$

$$T_{subsequent\ inetrval} = T_{propagation\ to\ 1.5mm} - T_{propagation\ to\ 0.5mm} = 1190 \text{ cycles.} \tag{6-9}$$

The interval of first inspection and average intervals after 2nd inspection (3350 and 1179) of the non-periodic scheme is slightly smaller than the values (3639 and 1190) from RFC but similar.

6.3.2. The effect of random failure

Normal RFC procedure does not consider the random failure which is less probability comparing with failure due to fatigue cracks. This is a reasonable process but still may cause minor error. The non-periodic approach is applied omitting the random failure as it was done in RFC. The reliability of single element during the service life is shown in Fig. 6-10. The intervals are slightly longer than the ones shown in Fig. 6-8 and Fig. 6-9.

The interval of first inspection and average intervals after 2nd inspection (3560 and 1199) when omitting random failure is very similar with the values (3639 and 1190) from RFC.

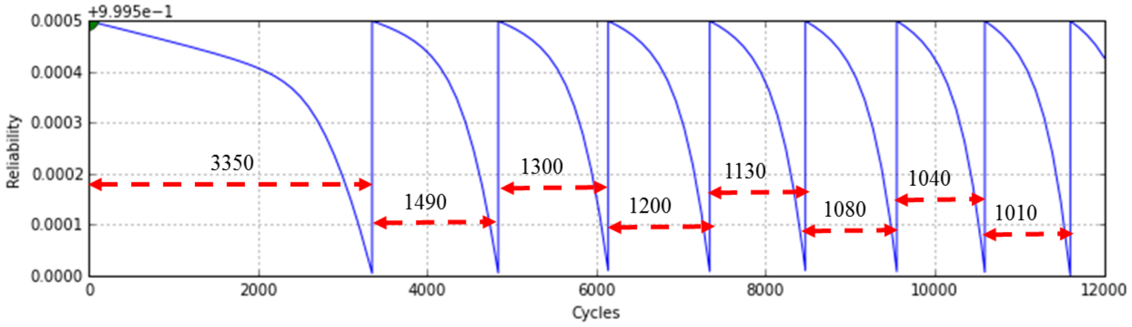


Fig. 6-8 Reliability of single element (1 element system, true value)

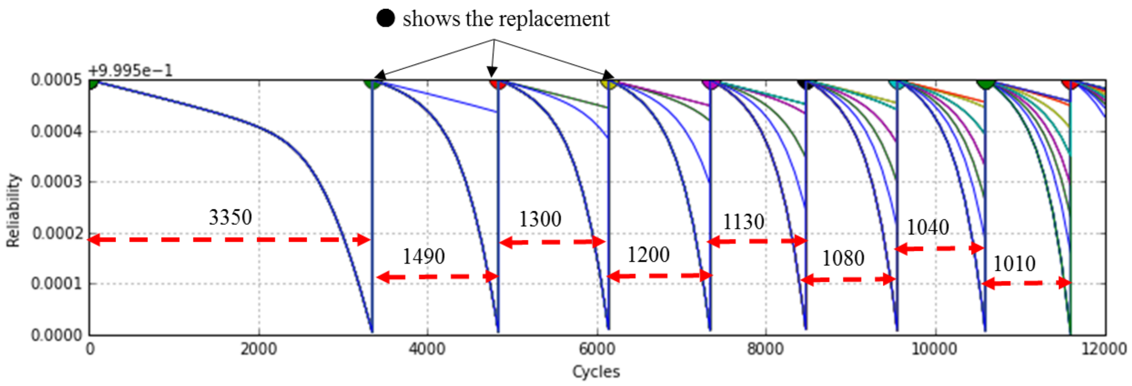


Fig. 6-9 Reliability of single element (500 element system, true value)

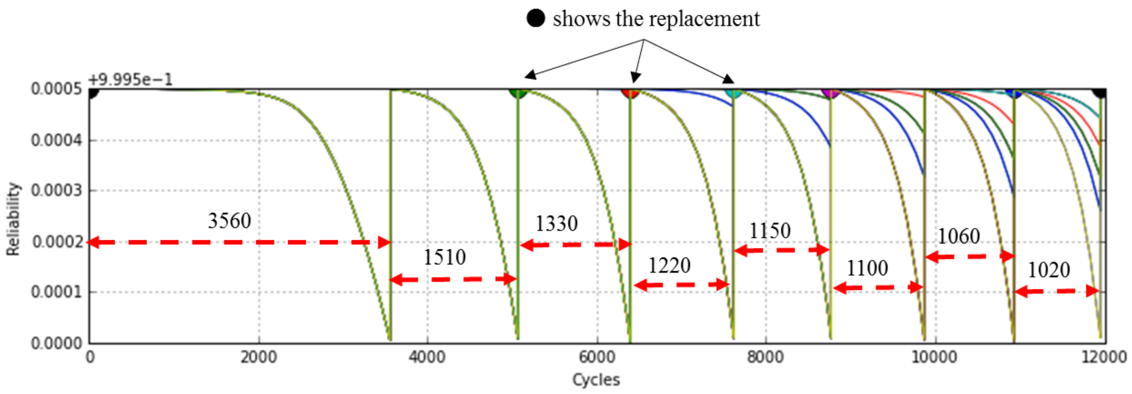


Fig. 6-10 Reliability of single element (500 elements, true value, ignore random failure)

6.3.3. The effect of probability of detection

The effect of probability of detection is discussed in this section. In normal RFC procedure, the POD is assumed to be 100% for the sake of simplification. The non-periodic scheme is applied for the cases that POD of 0.5mm crack increase from 0.63 to 0.94 (Parameter *d* changes from 0.5 to 0.4).

Simulation results of the reliability of single element during the service life is shown in Fig. 6-11. The first interval keeps the same while the subsequent intervals increase a lot and only six inspections are needed for the whole service life. The results shows that neglecting the POD will cause big error for the subsequent inspection intervals.

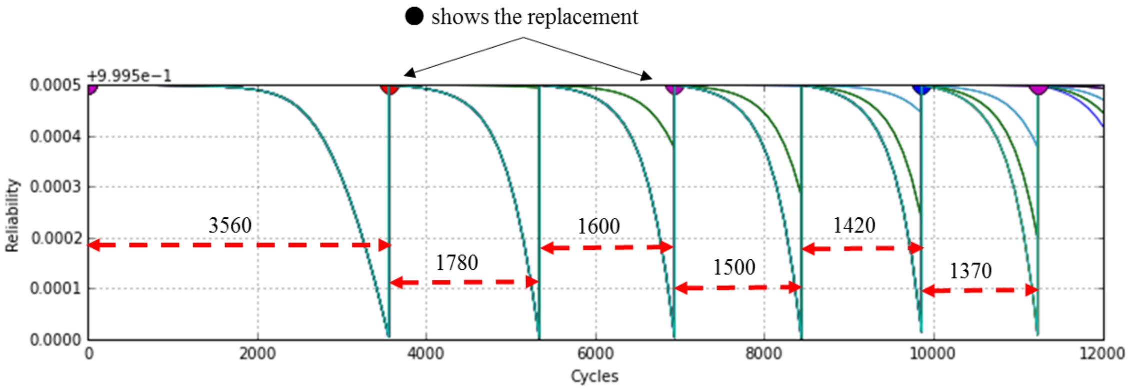


Fig. 6-11 Reliability of single element (500 element, true value, ignore random failure, higher POD)

6.4. Non-periodic inspection intervals with an uncertain parameter

As describe above, only the parameter *c* in the equation of crack propagation is considered as uncertain. Total number of elements is 500, which gives more information to the Bayesian updating comparing with chapter 3 and 4. This makes the Bayesian updating possible to converge faster and better to the true value.

One simulation result is shown in Fig. 6-12 and Fig. 6-13, where Fig. 6-12 is the change of distribution function for parameter *c*, Fig. 6-13 is the reliability of single element during the service life. The inspection time and intervals are listed in Table 6-3. Most of the intervals become smaller as a function of time. Inspection intervals become similar in the latter half of the service life.

Another simulation results is shown in Fig. 6-14 and Fig. 6-15, and the inspection time and intervals are listed in Table 6-4. This is a unique case in which there is no crack found in 2nd to 4th inspections. The distribution function is updated as that the parameter *c* is more likely be a small value. Cracks may potentially exist in the components because of the slow propagation, thus inspection intervals is predicted to be smaller than in schedule 1. The interval become longer when some cracks found in 5th inspection and distribution function move to the true value gradually.

Table 6-3 Inspection schedule 1 (uncertain parameter *c*)

Inspection no.	Inspection time (cycles)	Inspection interval (cycles)
1	3350	3350
2	4690	1340
3	5980	1290
4	7250	1270
5	8340	1090
6	9520	1180
7	10640	1120
8	11700	1060

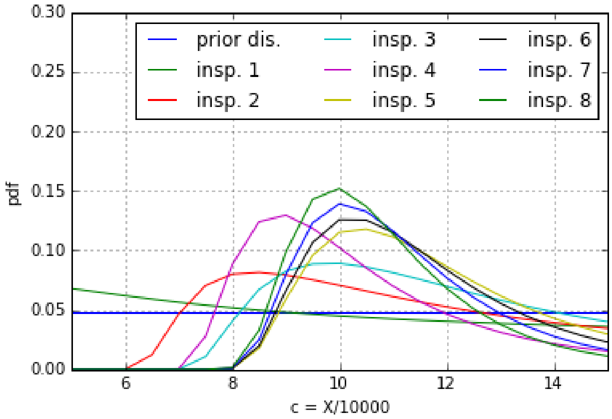


Fig. 6-12 Change of probability distribution function in scheme 1 (uncertain parameter *c*)

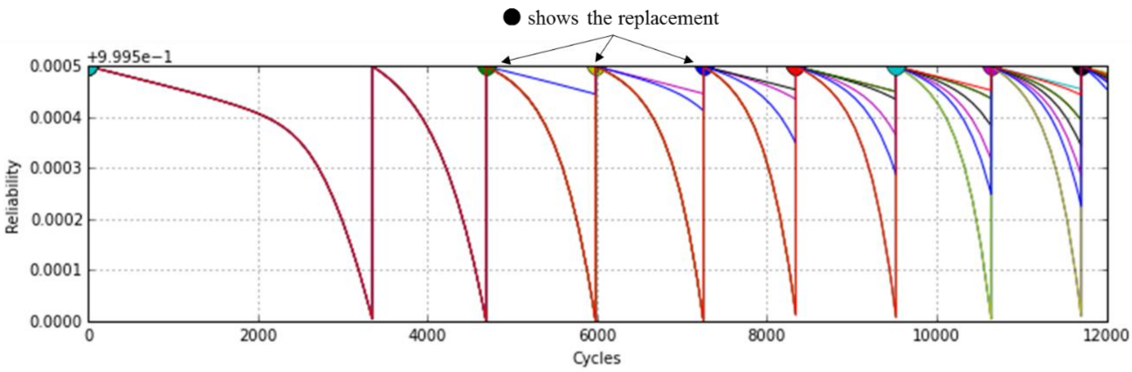


Fig. 6-13 Reliability of single element in inspection scheme 1 (500 elements)

Table 6-4 Inspection schedule 2 (uncertain parameter *c*)

Inspection no.	Inspection time (cycles)	Inspection interval (cycles)
1	3350	3350
2	4630	1280
3	5570	940
4	6390	820
5	7130	740
6	8140	1010
7	9260	1120
8	10500	1240
9	11600	1100

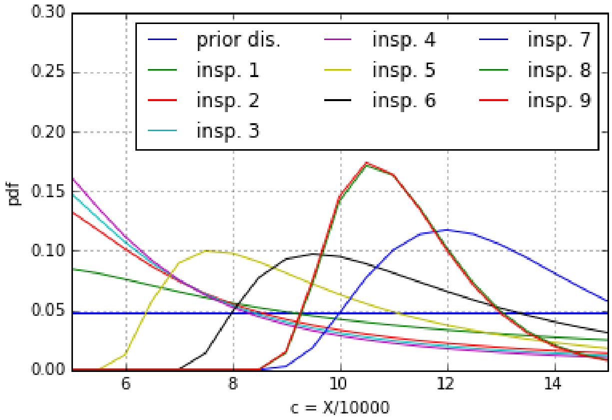


Fig. 6-14 Change of probability distribution function in scheme 2 (uncertain parameter *c*)

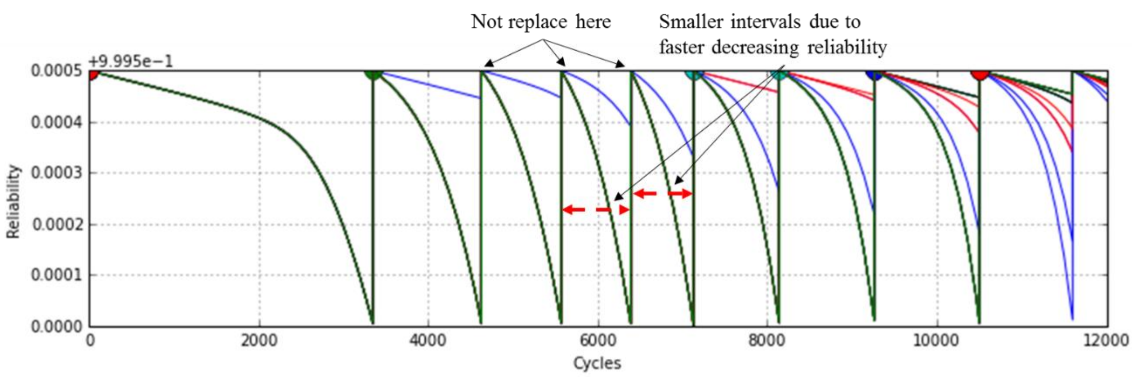


Fig. 6-15 Reliability of single element in inspection scheme 2 (500 elements)

6.5. Statistical analysis results

Two examples of simulation results are shown in former section. The first inspection interval, only depended on the initiation of fatigue cracks, keep a constant value no matter how the uncertain parameter (concern to the propagation) is. Following inspection intervals are mainly depended on the updating of the uncertain parameter *c*. This is a reasonable consequence because the parameter *c* is unknown and only information from inspection results are considered available.

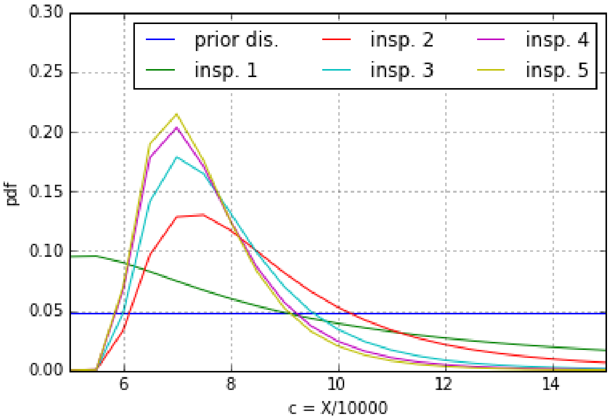
Bayesian updating with three different true values (0.7E-3, 1.0E-3 and 1.3E-3) of parameter *c* are shown in Fig. 6-16. Both three cases converge close to the true values eventually. Different from the examples shown in chapter 3 and 4, the Bayesian updating of parameter *c* converges to the true values faster and better. The eventual peak values are also higher which means the values are more concentrated to the true values. This is mainly results from the fact that the number of elements (500) is bigger and more information are obtained from each inspection.

Statistical analysis are performed treating parameter *c* as uncertain but actually it has a true value (for example, 0.7E-3, 1.0E-3 and 1.3E-3). This is a practical truth that the crack propagation may follow a kind of function with a certain parameter which is unknown beforehand. With different true values of parameter *c*, the RFC procedure is applied, the first inspection time and sequent inspection intervals (RTS intervals, return to service intervals) can be computed as eq. (6-8) and (6-9) shown. The non-periodic inspection intervals are also estimated by presented approach with the true values of parameter is known beforehand. The non-periodic inspection scheme is fixed when parameter is fixed.

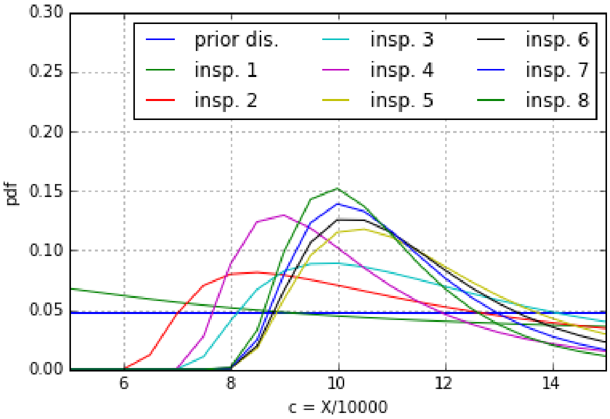
Results of 9 cases (3 cases of RFC procedure, 3 cases of non-periodic inspection with true value, and 3 cases of non-periodic inspection using Bayesian method) are shown in Table 6-5. The time of first inspection, the interval for 2^{nd} inspection, the average interval from 2^{nd} inspection to last inspection and the number of inspection are compared.

First, the results by RFC procedure and by presented approach are very similar. The most important point is that the average interval after 2^{nd} inspection is nearly the same by either methods. It should be noted that the interval for 2^{nd} inspection is longer in the cases of non-periodic inspection, and eventually reach the same average interval as RFC procedure. This is a logical results considering the aging of un-replaced components. The similarity between the results of two methods also proves the practicality of presented approach.

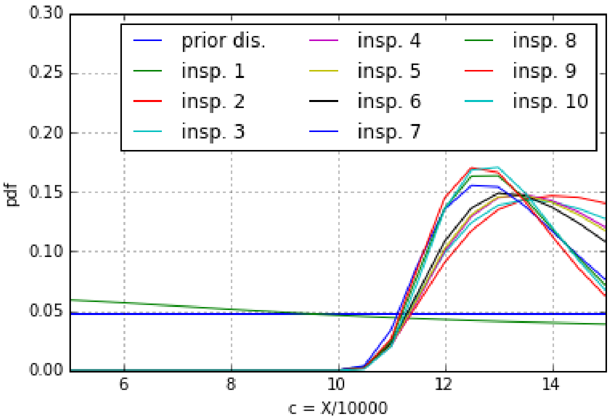
Second, the results by presented approach when treating parameter *c* as uncertain values and fixed values are also very similar. In other words, the Bayesian method solves the uncertainty of the parameter and gives a result similar to that when the value of the parameter is known. The standard deviation of inspection intervals is around 10% for all cases. It may be considered as a practical level with acceptable error.



(a) True value 0.7E-3



(b) True value 1.0E-3



(c) True value 1.3E-3

Fig. 6-16 Change of probability distribution for different true value

Finally, as shown in Table 6-5, RFC procedure and non-periodic inspection using a true value of parameter are very sensitive to the value itself. That is to say, when a wrong value of parameter is used, a wrong scheme will be obtained. On the other hand, Bayesian updating which treats this parameter as uncertain will always leads to a proper scheme. Comparisons of average interval from 2nd inspection (RTS interval in the RFC procedure) when the right value of parameter *c* is 0.7E-3, 1.0E-3 and 1.3E-3 separately are shown in Fig. 6-17, Fig. 6-18 and Fig. 6-19. Wrong prior knowledge of the value of parameter leads to wrong answer. On the other hand, the Bayesian approach gives a good results which error is less than 5% (assuming RFC with right value is the right answer).

6.6. Summary

An example of application to gas turbine engine components is shown in this chapter. Contents and results are summarized as follows:

1. Appropriate modifications are done in order to apply this approach to inspection scheme of engine components. Parameters are decided according to a reference article.
2. Inspection scheme similar with a RFC procedure can be obtained by presented approach, and more flexibility by including random failure, POD and uncertainty of crack propagation parameter is available.
3. The Bayesian approach is an advanced method for optimization of non-periodic inspection intervals. It can give a same or even better scheme than normal RFC method and shows great robustness against the prior knowledge.

Table 6-5 Statistical results of different inspection schemes (1000 samples)

Conditions	Time of First inspection	Inspection interval for 2nd inspection		Average interval from 2nd inspection		Number of inspections	
		Mean	SD	Mean	SD	Mean	SD
RFC: *c* = 0.7E-3	4385	1701	---	1701	---	5	---
True: *c* = 0.7E-3	4210	1810	0	1502	0	6	0
Uncertain *c* (0.7E-3)	4210	1952.3	97.4	1615.5	177.8	5.45	0.60
RFC: *c* = 1.0E-3	3639	1190	---	1190	---	8	---
True: *c* = 0.7E-3	3350	1490	0	1179	0	8	0
Uncertain *c* (1.0E-3)	3350	1400.1	106.5	1188.4	148.3	7.94	1.05
RFC: *c* = 1.3E-3	3237	916	---	916	---	10	---
True: *c* = 0.7E-3	2880	1290	0	981	0	10	0
Uncertain *c* (1.3E-3)	2880	1097.7	96.4	929.5	96.9	10.46	1.22

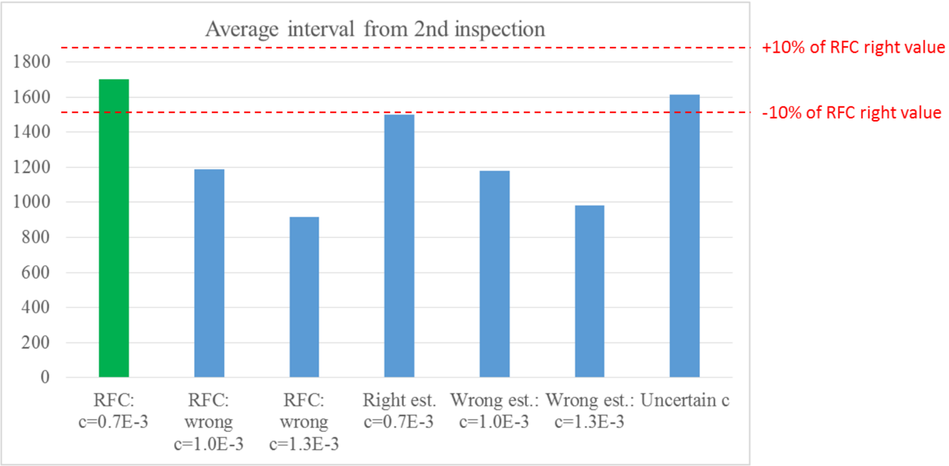


Fig. 6-17 Comparison of average interval from 2nd inspection (right value of *c*: 0.7E-3)

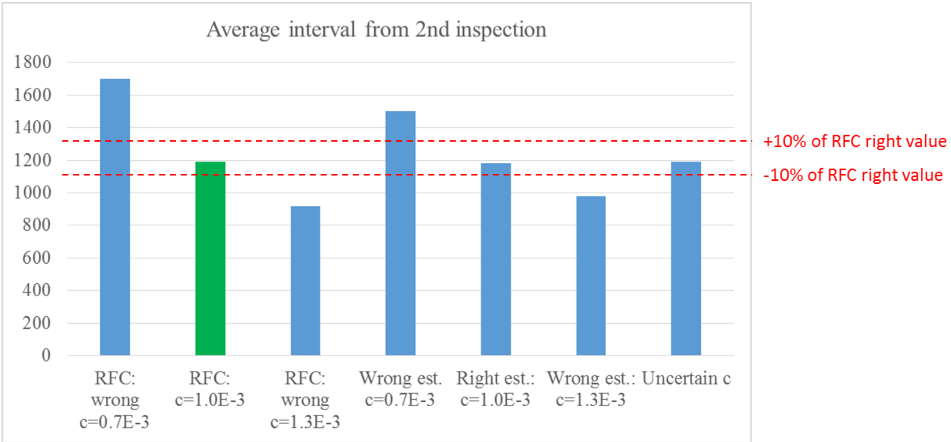


Fig. 6-18 Comparison of average interval from 2nd inspection (right value of *c*: 1.0E-3)

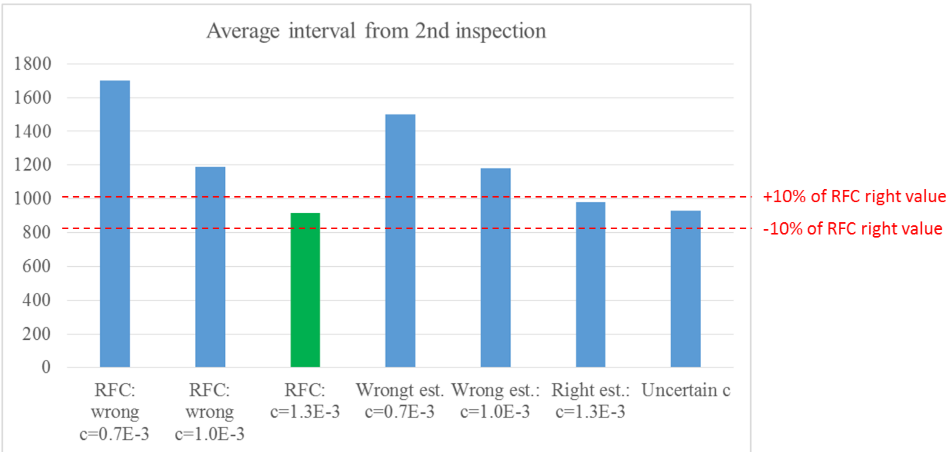


Fig. 6-19 Comparison of average interval from 2nd inspection (right value of *c*: 1.3E-3)

7. Conclusion

An advanced computational method for optimization of non-periodic inspection intervals for aging infrastructure is presented in this report. Fatigue is one of the most important problems of aging infrastructure subjected to random dynamic loads. Fatigue damage is considered to initiate in structural element and continues by crack propagation, resulting in strength degradation.

The whole or part of the aging infrastructure is refereed as a system which consists of a specific number of elements. The system is modeled by functional forms including equations for fatigue crack initiation, crack propagation, probability of detection, failure rate and probability of safety. All possible events and their probability are considered in order to estimate the reliability of a certain element at any specific time.

Some parameters of these functions are considered as possible sources of uncertainty. Bayesian method is applied in order to solve the problem of these uncertainties, and these parameters are modified according to the information from inspection results. The reliability of the entire system considering a distribute function of uncertain parameters is calculated by an integral over whole variable space.

The optimization of non-periodic inspection intervals is performed by computing the reliability step by step and find the maximum value satisfying that the reliability maintains above a pre-specified level. The advanced computational method introduced here is a combination of probabilistic analysis and Bayesian updating. Any simulation results by this method are casual results due to the uncertainty of crack initiation, uncertainty of crack detection, and the uncertainty of parameters as well. Statistical analysis is performed to evaluate the effect of the advanced computational method.

Discussions and conclusions in this report are summarized as follows:

1) Non-periodic inspection scheme using probabilistic method

   The non-periodic inspection scheme is optimized by an advanced probabilistic analysis method. The non-periodic inspection scheme is obviously superior to periodic inspection because of higher reliability and less inspections. Results show that this advanced approach can reduced inspection cost and at the same time maintain the estimated reliability above a required level. The presented approach is applicable to optimization of inspection intervals because of its high reliability.

2) Bayesian method for uncertain parameters

Bayesian method is applied to deal with the uncertainty of parameters. By Bayesian updating, the uncertain parameters can be estimated appropriately and reasonable inspection interval is scheduled. Wrong estimated value of parameter will result in wrong inspection scheme which cause low reliability or high cost. Bayesian updating can revise wrong prior knowledge and more reasonable inspection scheme is obtained. The selection of uncertain parameters is important and single uncertain parameter is preferred. Bayesian method works better if more information is available.

3) Application to turbine engine components

An application example for turbine engine components is shown in this report. Appropriate modifications and parameter selections are performed according to knowledge in this works and reference. Inspection scheme similar to a RFC procedure can be obtained, and more flexibility by including random failure, POD and uncertainty of crack propagation parameter is available. The advanced Bayesian approach can optimized the non-periodic inspection interval when the parameter of crack propagation function is unknown, while the normal RFC procedure meets difficulties. The Bayesian approach gives a same or even better scheme than normal RFC method and shows great robustness against the prior knowledge.

Reference

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